JAWAHARLAL NEHRU COLLEGE, BOKO

DEPARTMENT OF MATHEMATICS

A project submitted for partial requirement of fulfillment of the studies of B.Sc (Hons.) in mathematics, 6th semester 2021.

TOPIC

"A STUDY OF CO-ORDINATE GEOMETRY AND ITS APPLICATION IN REAL FIELD".

BY

KAKULI RABHA

ROLL NO: US-181-019-0043

REGISTRATION NO: 18034411

CERTIFICATE

This is to certify that the work contained in the project entitled "**A study of co-ordinate geometry and its application in real field**" submitted by Kakuli Rabha (Regd. No: 18034411) for the award of the degree of Bachelor of Science, under Gauhati University carried out by her under my direct supervision and guidance.

I considered that the project has reached the standards and fulfilling the requirements of the rules and regulation related to the nature of the degree.

Place: Jawaharlal Nehru College, Boko

Date: 23-08-2021

(Dr. Dhiraj Kumar Das)

HOD of Mathematics

Jawaharlal Nehru College, Boko

Declaration

I hereby declare that the featuring of this project is a result of my own research which has been carried out sincerely under the guidance of our respected Professor **Dr. Dhiraj Kumar Das** of **Department of Mathematics**, Jawaharlal Nehru College, Boko. I would like to declare that no part of this project has been previously submitted to this college to the best of knowledge.

Place: Jawaharlal Nehru College, Boko

Date: 23-08-2021

Kakuli Rabha

(Signature of student)

Acknowledgement

I do hereby thank my mentor **Dr. Dhiraj Kumar Das** for offering assistance on my project. I also take this opportunity to thank all the teachers of Department of Mathematics of Jawaharlal Nehru College, Boko for being always available.

I would also like to thank and express my deep appreciation to my well-wishers for their personal support and encouragement in doing this project.

ABSTRACT :

Co-ordinate geometry is considered to be one of the most interesting concepts of mathematics. Co-ordinate geometry describes the link between geometry and algebra through graphs involving curves and lines. It is a part of geometry where the position of points on the plane is described using an ordered pair of numbers. In this project, we will learn about the concepts of co-ordinate geometry along with its formulas and its application in real field.

KEYWORS : geometry, algebra, co-ordinate, graph.

CONTENTS

1. INTRODUCTION :

Co-ordinate geometry refers to the study of algebraic equations on graph. The method of describing the location of points in this way was proposed by the French mathematician René Descartes (1596 - 1650) who developed it in the 17th century. Descartes who liked to stay in bed until late was watching a fly on the ceiling from his bed. He wondered how to best describe the fly's location and decided that one of the corners of the ceiling could be used as a reference point. He imagine the ceiling as a rectangle drawn on a piece of paper, taking the left bottom corner as the reference point, now can specify the location of the fly by measuring how far we need to go in horizon direction and in the vertical direction to get to it. These two numbers are the fly's co-ordinate Every pair of co-ordinates specifies a unique point on the ceiling.

He proposed further that curves lines could be described by equations using this technique, thus being the first to link algebra and geometry. In honor of his work the co-ordinates of a point are often referred to as its Cartesian co-ordinates and the co-ordinate plane as the Cartesian co-ordinate plane.

What is Co-ordinate geometry?

A co-ordinate geometry is a branch of geometry where the position of points on the plane is define with the help of an ordered pair of numbers also known as co-ordinates. Co-ordinate geometry is also known as analytical geometry.

2. ANALYTICAL GEOMETRY OF ONE DIMENSION :

A one dimensional co-ordinate system is defined by its origin and a single basis vector that defines the positive direction of the co-ordinate axis (x-axis). The co-ordinates of any point in such a system are determined by a single real number. A line with a chosen Cartesian system is a number line. Every real number has a unique location on the line.

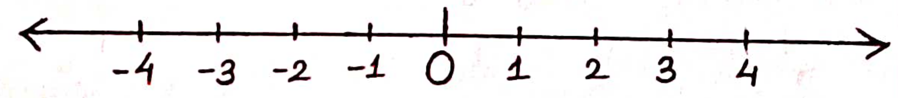


Fig: Number Line

For example : To ~~locat~~ describe the location of a train along a straight section of track that runs in the East-West direction and taking the origin i.e the station. Then we can describe the position of the train by specifying how far it is from the station using a single real number, say 'x'.

3. ANALYTICAL GEOMETRY OF TWO DIMENSIONS :

3.1 Cartesian coordinates:

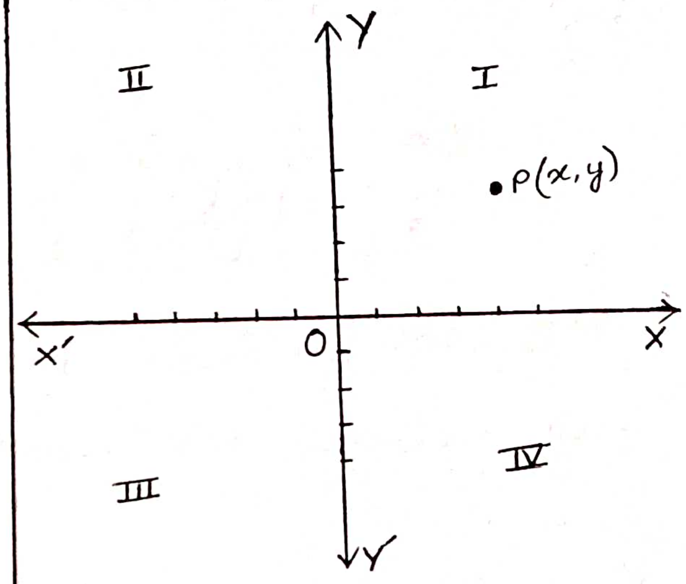


Fig: Two Dimension Cartesian System

Coordinates are a set of values which helps to show position of a point in the coordinate plane. A coordinate plane is a two-dimension surface formed by two number lines. One number line is horizontal and is called the x-axis (XOX'). The other number line is vertical number line and is called the y-axis (YOY'). The two axes meet at a point called origin. We can use the coordinate plane to graph points, line etc. When the origin is in the centre of the plane, they divide it into four areas called quadrants I, II, III, IV. The co-ordinates of any point, P on the Cartesian plane are generally denoted by (x, y).

Let us take an example: In a class we need to find the position of roll no. 1 say Jaminur by using two independent informations.

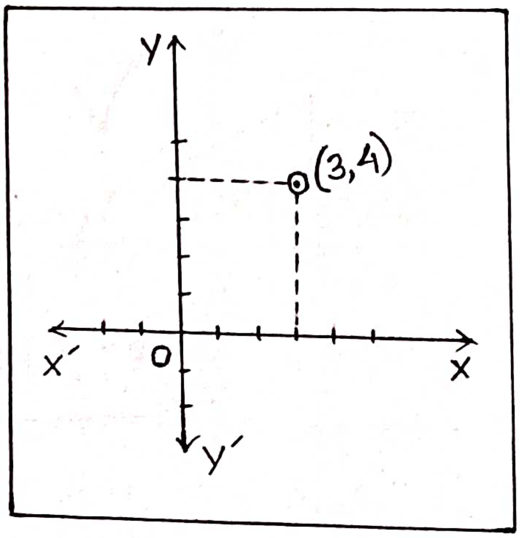
i) The column in which he sits

ii) The row in which he sits

If he is sitting on the desk in the 5^{th} column and 6^{th} row then his position can be denoted by (5, 6)

Locating the points in a Cartesian plane:

Let us consider a point (3, 4) is 4 units away from the x-axis measured along the positive Y-axis and 3 units away from Y-axis measured along the positive X-axis.



The points having x-coordinate as '0' lie on the Y-axis and points having y-coordinate as '0' lies on the X-axis. For example, the points (2,0) & (0,5) lies on X-axis and Y-axis respectively.

3.2 : Distance between two Points :

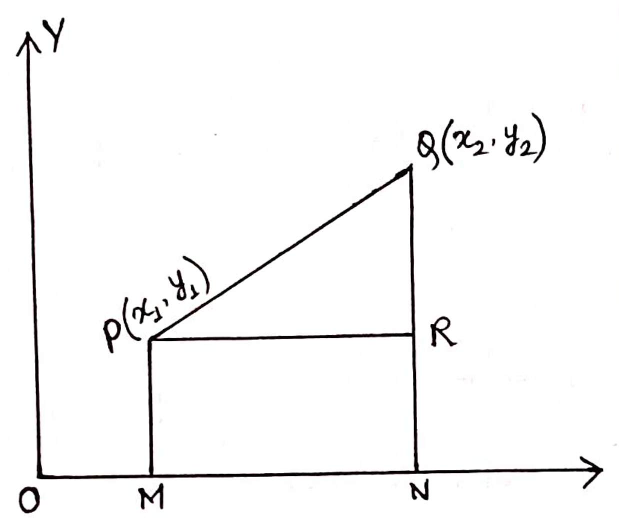


Fig. : Distance formula

Let the coordinates of two points P and Q be (x_1, y_1) and (x_2, y_2) referred to rectangular axes OX and OY. PM and QN are perpendiculars to OX. PR is perpendicular to QN

Now $OM = x_1$, $MP = y_1$, $ON = x_2$, $NQ = y_2$

From the figure,

$$PR = MN = ON - OM = x_2 - x_1$$

$$RQ = NQ - NR = NQ - MP = y_2 - y_1$$

From the right-angled triangle PRQ

$$PQ^2 = PR^2 + RQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$, which is the required distance between two points

3.3 Midpoint Theorem :

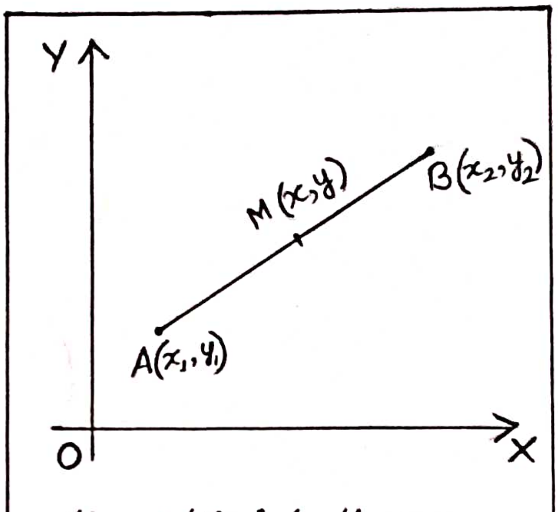


Fig : Midpoint theorem

Let us consider two points A and B having coordinates to be (x_1, y_1) and (x_2, y_2) respectively. Let $M(x, y)$ be the mid-point lying on the line AB.

The coordinates of the point M is given as

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

3.4 Section Formula :

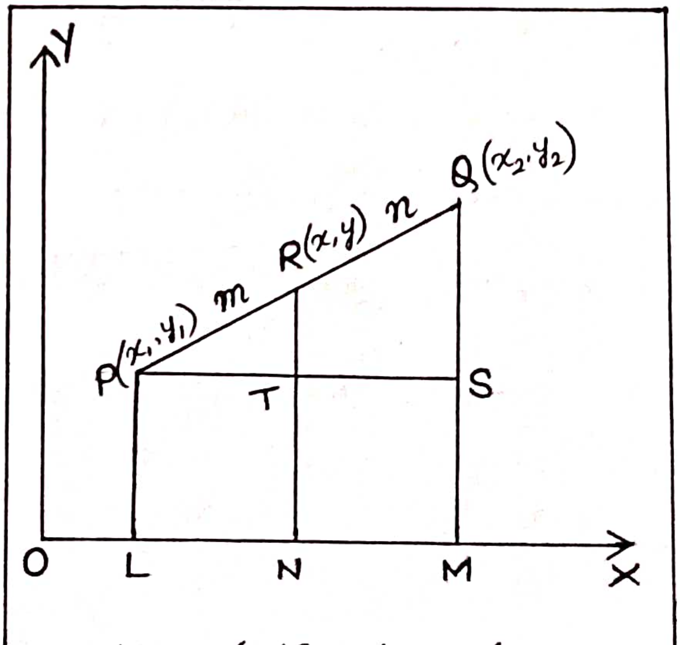


Fig : Section formula

Let us consider a line P and Q having coordinates (x_1, y_1) and (x_2, y_2) respectively. Let R divide the segment PQ in the ratio $m:n$. The coordinates of the point R is given as

* When the ratio $m:n$ is internal :

$$\left(\frac{mx_2 + nx_1}{m+n} , \frac{my_2 + my_1}{m+n}\right)$$

* When the ratio $m:n$ is external :

$$\left(\frac{mx_2 - nx_1}{m-n} , \frac{my_2 - ny_1}{m-n}\right)$$

3.5 Area of a triangle :

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

AL, BM and CN are perpendicular to OX

Now ΔABC = trapezium ABML + trapezium ALNC − trapezium BMNC.

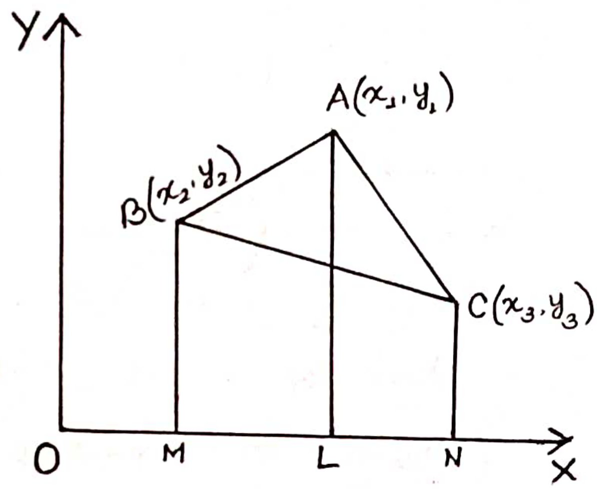


Fig : Area of a triangle on a Cartesian plane.

Thus

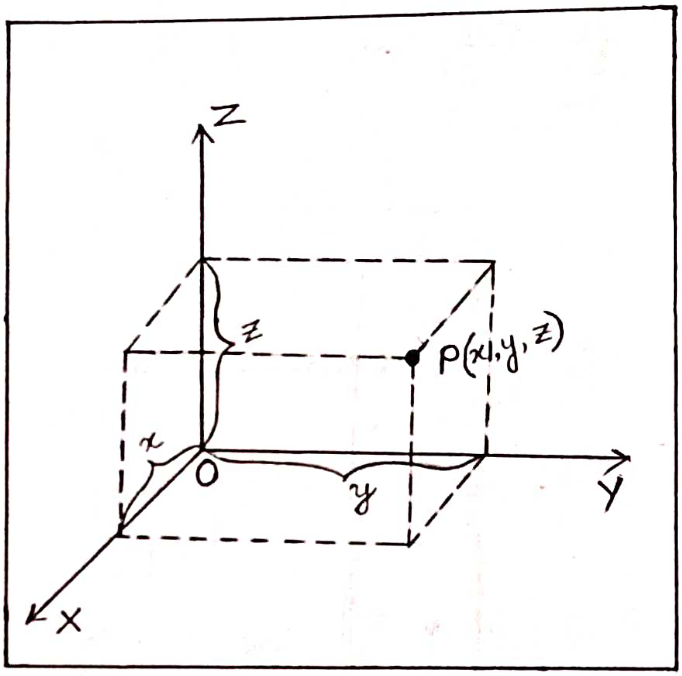
$$\Delta ABC = \frac{1}{2}\left[(y_2+y_1)(x_1-x_2) + (y_1+y_3)(x_3-x_1) - (y_2+y_1)(x_3+x_2)\right]$$

$$= \frac{1}{2}\left[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right]$$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

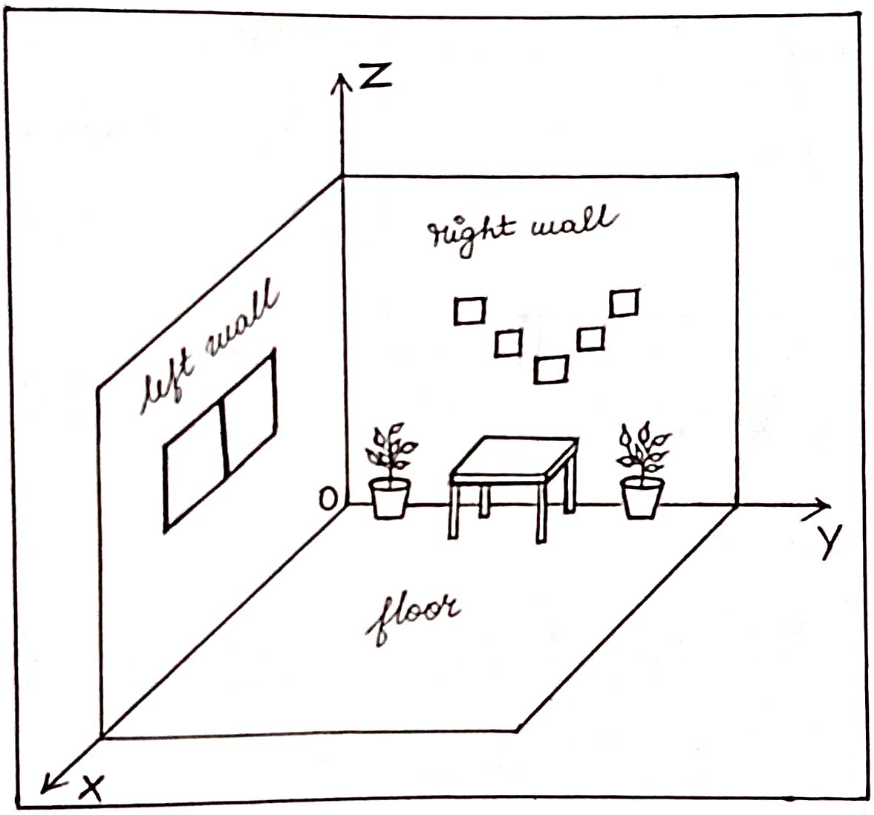
4. ANALYTICAL GEOMETRY OF THREE DIMENSIONS :

4.1 Cartesian Coordinate :



A three dimensional Cartesian coordinate system is formed by a point called origin denoted by O and a basis consisting of three mutually perpendicular vectors. These vectors define the three co-ordinate axes: the x-axis, y-axis and z-axis. They also known as the abscissa, ordinate and applicate axis respectively. The co-ordinate of any point in space are determined by 3 real numbers: x, y, z. The three co-ordinate axes determine three co-ordinate planes. The xy-plane is the plane that contains the x-axis and y-axis; the yz-plane contains the y-axis and z-axis; the xz-plane contains the x- and z-axes. These three co-ordinate planes divide space into eight parts called octants.

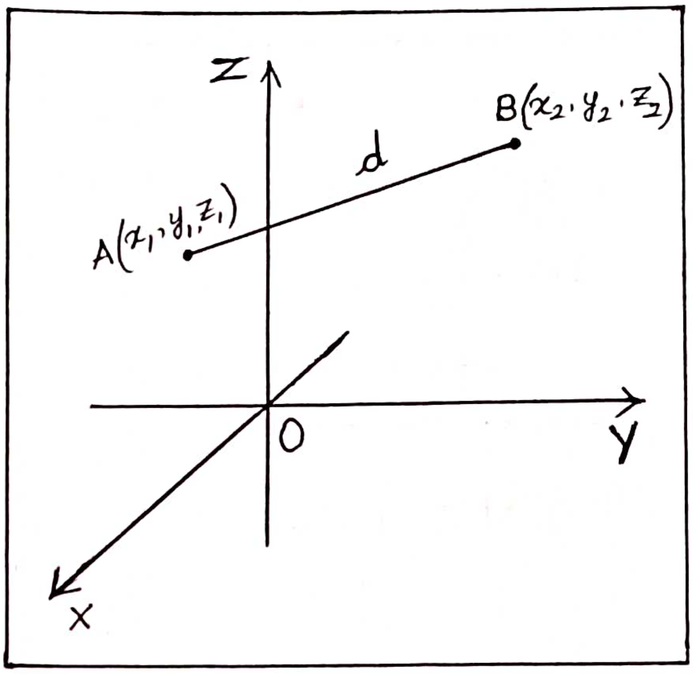
Let us understand three dimensional co-ordinate system more clearly from our surrounding. Let us take the example of our room.



Considering one corner as the origin and the wall on your left is the xz-plane and the wall on our right is the yz-plane. The floor is in the xy-plane. The x-axis runs along the intersection of the floor and the left wall, y-axis along that of the floor and the right wall. The z-axis runs up from the floor toward the ceiling along the intersection of the two walls.

4.2 Distance between two points :

The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space is determined by the formula



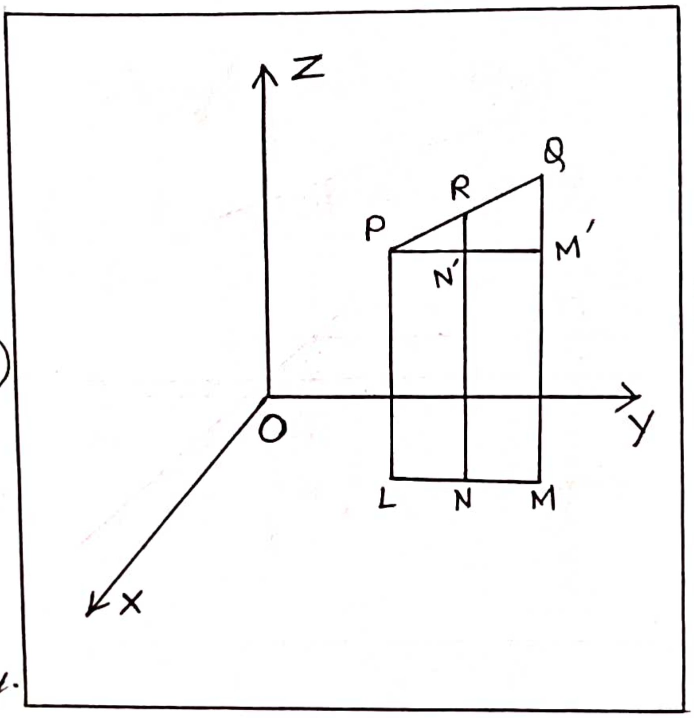
$$d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

4.3 Midpoint of the line segment AB :

The co-ordinates of the midpoint of the line segment AB are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

4.4 Section formula :



Let $R(x, y, z)$ divide the distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in the ratio $m:n$

* When the ratio $m:n$ divides internally.

The coordinates of R are

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}, \; z = \frac{mz_2 + nz_1}{m+n}$$

* When the ratio $m:n$ divides externally, the coordinates of R are

$$\cancel{x =} \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$$

4.5 Area of a triangle :

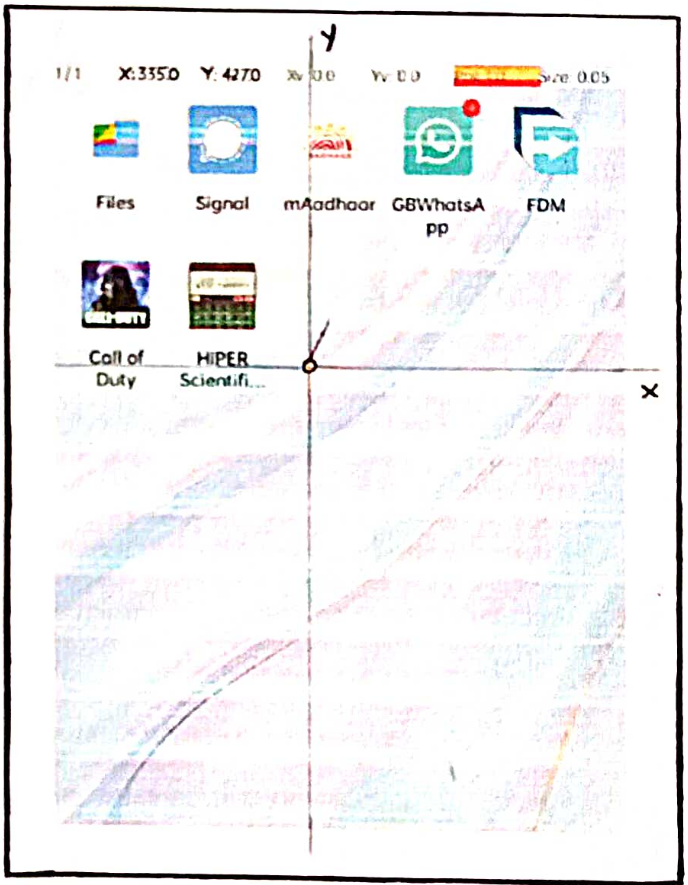
The area of a triangle with the vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is

$$S = \frac{1}{2}\left[\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}^2 + \begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix}^2 + \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2\right]^{\frac{1}{2}}$$

5. APPLICATION OF CO-ORDINATE GEOMETRY IN REAL FIELD :

5.1 In Digital world :

Co-ordinate geometry is applicable in computers or cell phones. The text file or the Portable Document Format (PDF) file which we open is itself an example of co-ordinate plane. The words or the images are written or modified with the use of co-ordinate geometry. Any PDF file, which contains text, image and different shapes, are placed according to the two-dimensional coordinate (x, y) system. The origin $(0, 0)$ point is located in the bottom left hand corner of the page. Horizontal or X co-ordinate increase to the rights and vertical or Y coordinate increase towards the top. Each words in a PDF page is bounded by a rectangle called 'Quad'. The Quad represents the (x, y) co-ordinate pairs for the 4 vertices of the rectangle bounding the word. So it is easy to find the location of any word in a PDF file.

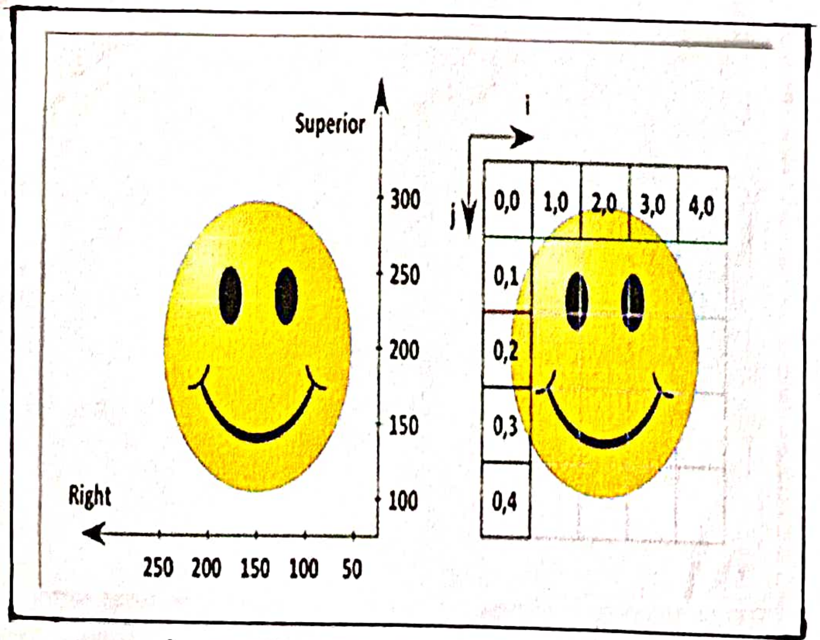


Touch screen input tracking in mobile.

Our smartphones screen also uses a Cartesian co-ordinate plane to track where we are touching on the screen. In the adjacent figure, we can see the touch screen of a smartphone with Touch screen input tracking. It shows the location of the ~~viewers~~ user's input ie $x = 335.0$ and $y = 427.0$

Co-ordinate geometry is also used for setting up items or windows on a computer screen. The entire screen on a computer is located in the first quadrant, where both x and y values are positive.

When we use paint brush or edit any image in computer the concepts of co-ordinate is used. On changing any shape or adding different colours changes the points on the co-ordinate plane.



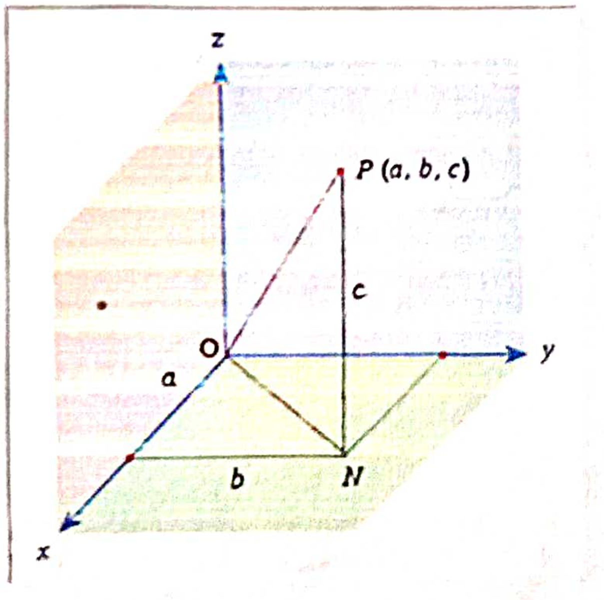
Paint brush

In digital camera, when we capture an image it uses the concept of co-ordinate geometry. It divides the whole image into many parts. Then it digitally store the co-ordinates and its colours.

Scanner and photo copying machine also uses co-ordinate geometry to produce the exact image of the original image provided to it.

5.2 Describing positions of any object :

The position of any object in the real world can be described using a co-ordinate geometry from its original point i.e the origin.



Position of a object

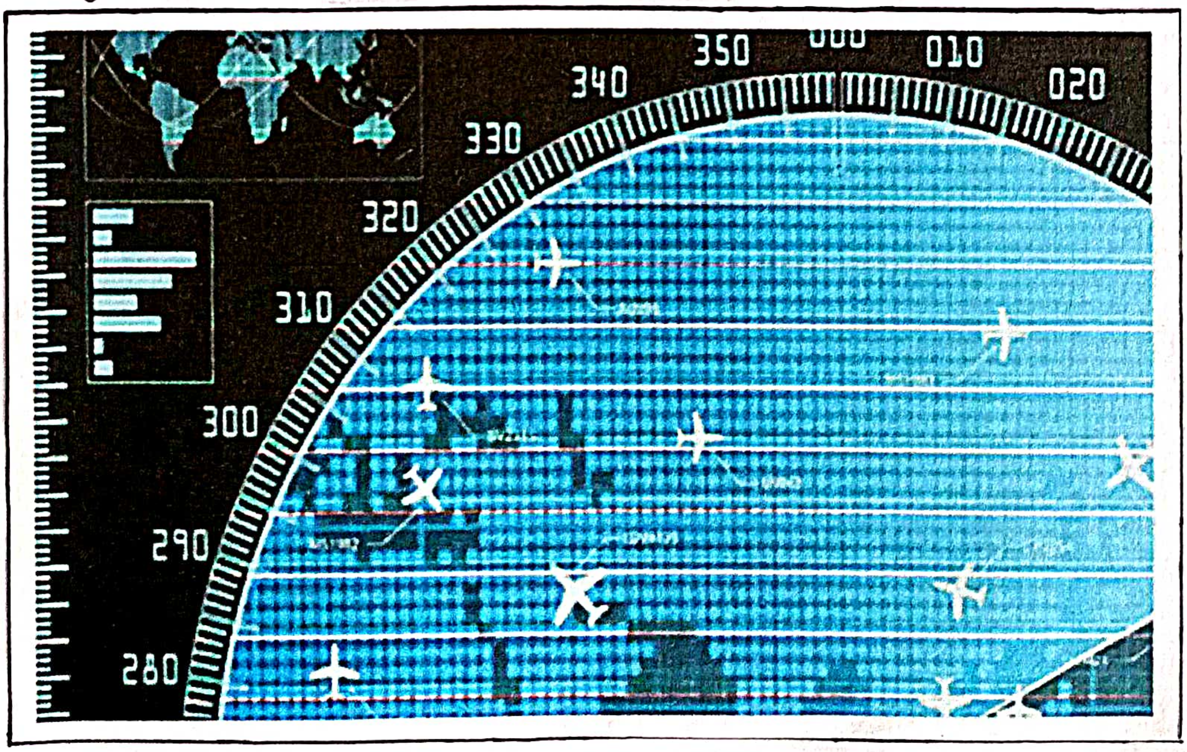
For example, we can find the position of our phone or a watch which is placed on the table. Let the horizontal distance of the table from the door is 10 metres and the vertical distance of the phone from the ground is 5 metres. If the room is 5 metres wide, then we can easily find the co-ordinates of the phone and thus can apply the distance formula to calculate its actual distance.

5.3 Location of air transport:

We all have seen the aeroplanes flying in the sky but might have not thought of how they actually reach the correct destination as the sky has no roads and no signboard. Actually all these air traffic is managed and regulated by using co-ordinate geometry.

How can we find the current location of an aircraft ?

All the air traffic is controlled by air traffic controller. A controller must know the location of every aircraft at any particular instant of time in the sky. Co-ordinates of any particular vehicle are used to describe its current location of aircraft. Even if an aircraft moves a small distance (up, down, forward or backward), its co-ordinates are updated in the system



Now imagine what if co-ordinate system doesnot exist. Pilots, aircraft controller, passengers in the flight, persons waiting for the flight all will not be able to get the location or position of aircraft. These will also definitely increase the chances of aircraft crashes. So co-ordinate system is one of the most important parts of air transport.

5.4 In Art and Design :

Interior Design :

The applications of co-ordinate geometry in daily life can also be found in interior design. Setting new items in an open space is done perfectly using the concepts of co-ordinate geometry. They use it to determine the best use of space, where to place different pieces of furniture.

Art :

The use of co-ordinate geometry in the field of ~~is~~ art is a very interesting topic. We can draw any figures using co-ordinate system, by plotting the co-ordinate points on the Cartesian plane. There are some figures given below -

i)

By plotting the above co-ordinates on the Cartesian plane we have created a picture of leaf. We have given some more figures below -

(1, -3), (5, -4), (4, -3), (9, 1), (7, 2), (8, 5), (5, 4), (5, 5), (3, 4), (4, 9), (2, 7), (0, 10), (-2, 7), (-4, 8), (-3, 3), (-5, 6), (-5, 4), (-8, 5), (-7, 2), (-9, 1), (-4, -3), (-5, -4), (0, -3), (2, -7), (2, -6), (1, -3)

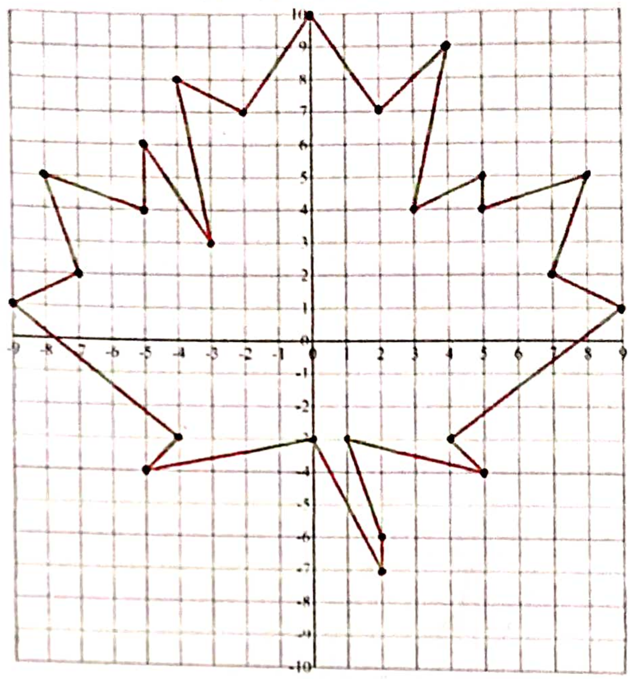


Fig: 1

(ii)

* (-5, -3), (-4, -5), (-3, -7), (-3, -6), (-2, -4), (-1, 0) - line ends
* (-7, 1), (-8, 4), (-9, 7) (-8, 10), (-7, 10), (-5, 9), (-3, 7), (-2, 6), (0, 7), (2, 6), (3, 7), (5, 9), (7, 10), (8, 10), (9, 7), (8, 4), (7, 1), (3, 4), (0, 1), (-3, 4) (-7, 1) - line ends
* (-5, 0), (-4, 0), (-2, -1), (-4, -1), (-5, 0) - line ends
* (5, -3), (4, -5), (3, -7) - line ends
* (7, 1), (8, -3), (9, -7), (8, -7), (7, -6) (4, -9), (1, -10), (-1, -10), (-4, -9), (-7, -6), (-8, -7), (-9, -6), (-8, -2), (-7, 1) - line ends
* (5, 0), (4, -1), (2, -1), (4, 0), (5, 0) line ends
* (1, 0), (2, -4), (3, -6), (3, -7), (1, -8) line ends

* (-4,-1), (-4,0), (-3,-1) - line ends {shade in}
* (-3,-7), (-1,-8), (1,-8), (2,-6), (1,-5), (-1,-5), (-2,-6), (-1,-8) - line ends {shade in}
* (4,6), (7,9), (7,5), (6,4), (4,5), (4,6) - line ends {shade in}
* (4,-1), (4,0), (3,-1) - line ends {shade in}
* (-7,4), (-7,9), (-4,6), (-4,5), (-6,3), (-7,4) - line ends {shade in}

By plotting the above co-ordinates we get the following fig.: 2

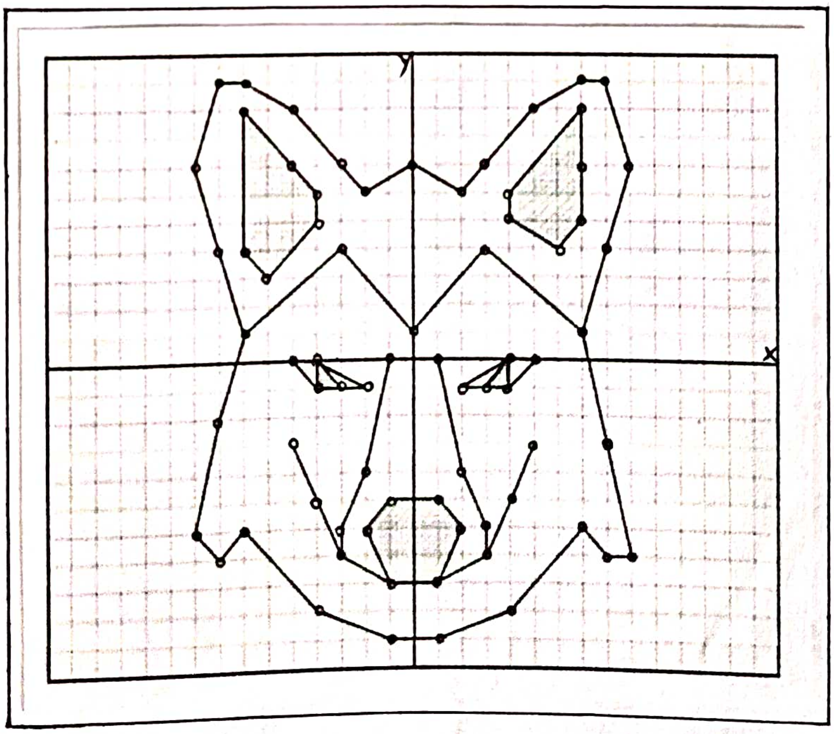


Fig.: 2

Here are two cartoon characters Mario and spongebob drawn on the Cartesian plane with the help of co-ordinate points.

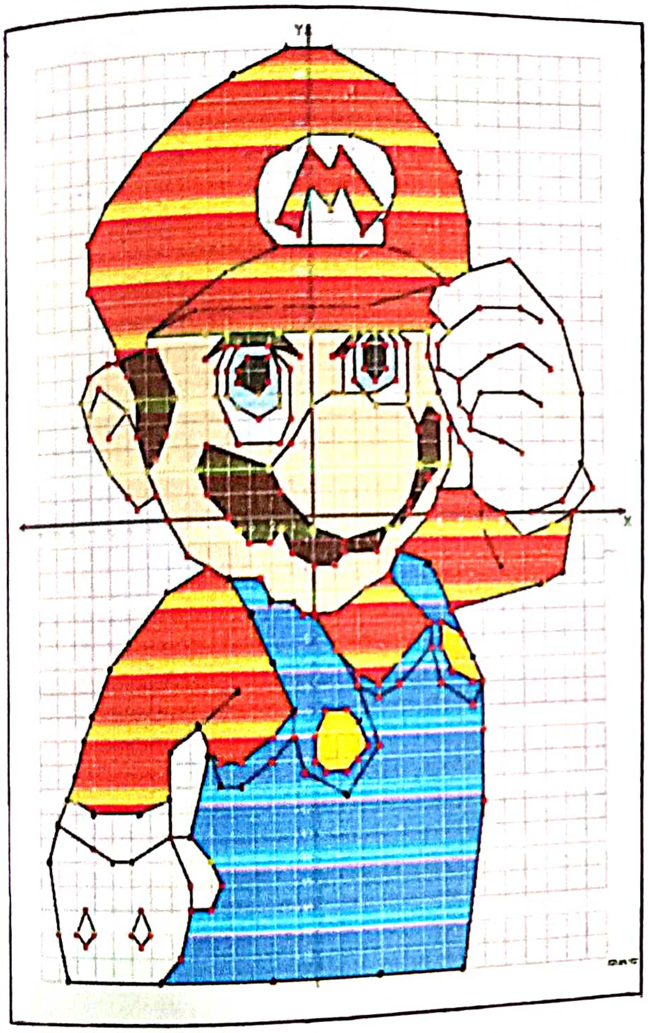


Fig : 3 - Mario

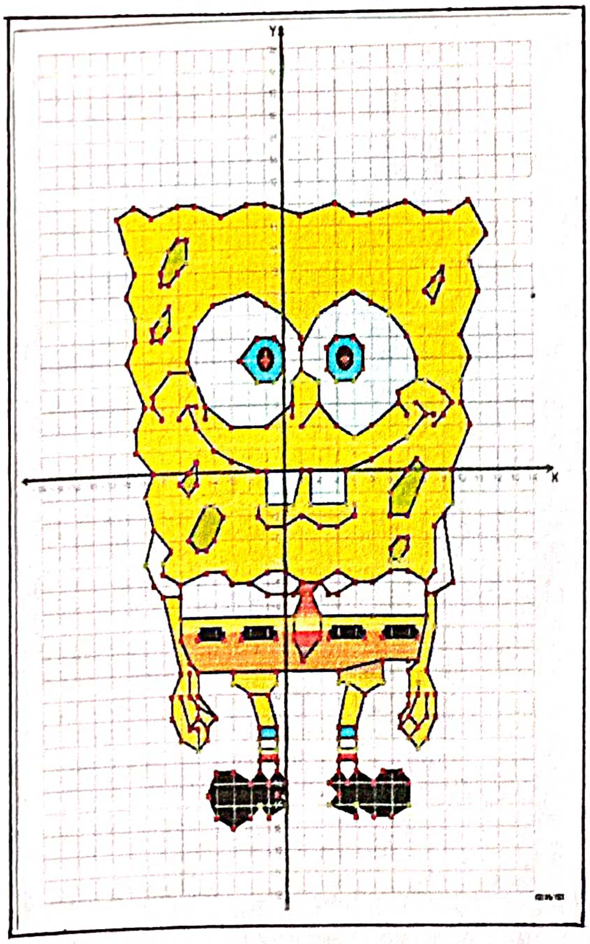


Fig.:4 - Spongebob

CONCLUSION :

Co-ordinate geometry deals with the study of algebric equations through graphs. It describes the linkage between geometry and algebra through graphs, involving curves and lines. In this project, the diffrent geometric dimensions are analized using the co-ordinate system. The use of co-ordinate geometry is put forth in real field with diffrent examples.

Co-ordinate geometry is considered as an important and interesting concept in mathematics. It has ~~many~~ valuable uses in the real world. Using co-ordinate geometry, it is possible to find the distance between any two objects and also finding the mid-point of a line etc.

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