TITLE OF THE PROJECT

A Study of Differentiation and its applications

The Project submitted
to the Department of Mathematics, Jawaharlal Nehru College, Boko
for the partial fulfillment for the award of
degree of Bachelor of Science,
Under
Gauhati University
Subject: Mathematics

By
Mr. Himangshu Kalita
Department of Mathematics
Registration No : 18034405
Roll No:US-181-019-0037

Under the Supervision of
Dr. Dhiraj Kumar Das

DEPARTMENT OF MATHEMATICS
JAWAHARLAL NEHRU COLLEGE, BOKO
KAMRUP:: ASSAM
PIN: 781123

CERTIFICATE

This is to certify that the work contained in the project entitled **" A study of differentiation and its applications"** submitted by **Sri Himangshu Kalita (Regd. No. 18034405)** for the award of the degree of Bachelor of Science , Under Gauhati University carried out by him under my direct supervision and guidance.

I considered that the project has reached the standards and fulfilling the requirements of the rules and regulation relating to the nature of the degree.

Date: 25/08/2021

Place: Boko

(Dr. Dhiraj Kumar Das)
HOD,Mathematics
Jawaharlal Nehru College, Boko

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I would also like to thank and express my deep appreciation to my well wishers for their personal support and encouragement in doing this project.

DECLARATION

I do hereby declare that the project work entitled ***"A study of Differentiation and its application"*** *submitted to the Department of Mathematics, Jawaharlal Nehru College, Boko. In partial fulfillment of the requirements for the Degree of Bachelor of Science is an original piece of work done by me under the guidance of* ***Dr. Dhiraj Kumar Das*** *and it has no resemblance to any other work done in any of the universities in the country. It is further declared that such help or source of information as has been availed of during the course of investigation has been duly acknowledged by me .*

Place : Boko

Date : 25/08/2021

Himangshu Kalita.

Himangshu Kalita

Roll No: US-181-019-0037

Department of Mathematics

Jawaharlal Nehru College, Boko

Abstract

This Project is written simply to illustrate on differentiation and there applications. The formation and classification of differentiation, the basic techniques of differentiation, list of derivatives and the basic applications of differentiation, which include optimization, motion and economic.

Keywords: Functions, Graphs, Tangent, Rate of change, limit, continuity, Differentiation, Related rates, optimization problem, maxima, minima.

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Chapter one

1.1 Introduction:

From the beginning of time man has been interested in the rate at which physical and non physical things change. Astronomers, physicists, chemists, engineers, business enterprises and industries strive to have accurate values of these parameters that change with time.

The Mathematician therefore devotes his time to understudy the concepts of rate of change. Rate of change gave birth to an aspect of calculus known as DIFFERENTIATION.

1.2 Scope of the Study:

This project work will give a vivid look at differentiation and its application.

It will state the fundamental of calculus, it shall also deal with limit and continuity.

1.3 Literature Review :

Calculus, historically known as infinitesimal calculus, is a mathematical discipline focused on limits, functions, derivation, integrals and infinite series.

Ideas leading up to the notion of function, derivatives and integral were developed through out the 17th century but the decisive step was made by Isaac Newton and Gottfried Leibniz, who provided independent and unified approaches to differentiation and derivatives. In the 19th century, calculus was put on a much more rigorous footing by mathematicians, such as Augustin Louis Cauchy (1789-1857), Bernhard Riemann and Karl Weierstrass. It was also during this period that the differentiation was generalized to Euclidean space and complex plane.

Chapter Two

2.0 Functions and Graphs

2.1 Functions ; Domain and Range

Functions are essential ingredients in the study of calculus. There absence means an incomplete study of this branch of mathematics.

Calculus by its basis is the notion of correspondence for example, the surface area (A) of a sphere relates to its radius by the formula $A = 4\pi r^3$.

It gives us an idea what a function is. Conclusion from this could imply that if the value of one quantity, say y depends on the value of another quantity say x, then for every value of x there corresponds one and only one value of y.

On this basis we say that y is a function of x.

Though, the same element of y may corresponds to different element of x. For example, two different books may have the same number of pages.

Definition 2.0: A function f from a set X to a set Y is a rule that assigns a unique value $f(x)$ in Y to each x in X.

The set X of all possible input values is called the domain of the function. The set of all output values of $f(x)$ as x varies throughout X is called the range of the function.

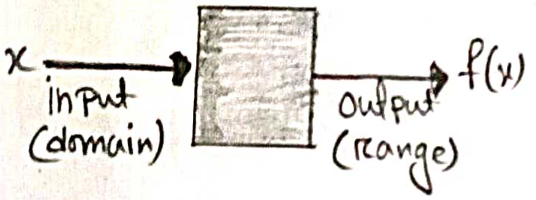


Fig 2.1: A function can be visualized as an input / output device.

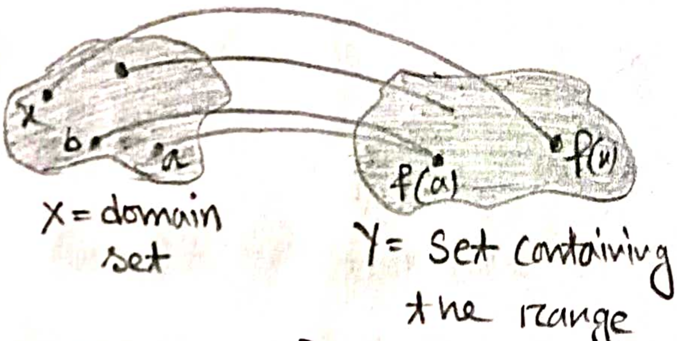


Fig 2.2: A function from a set X to a set Y assigns a unique element of Y to each element in X.

2.2 Graphs of Function:

If f is a function with domain X, its graph consists of the points in the cartesian plane whose coordinates are the input output pairs for f. In set notation, the graph is $\{x, f(x) \mid x \in X\}$

The graph of the function $f(x) = x+2$ is the set of points with coordinates (x, y) for which $y = x+2$. Its graph is the straight line sketched in Figure 2.3

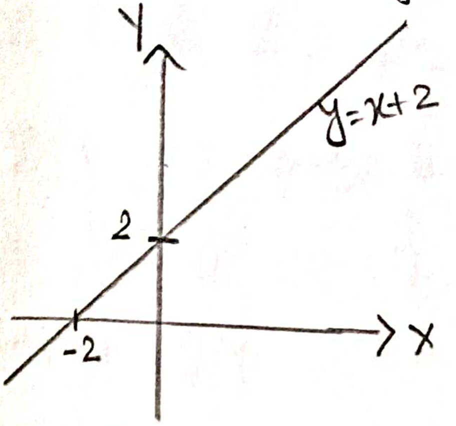
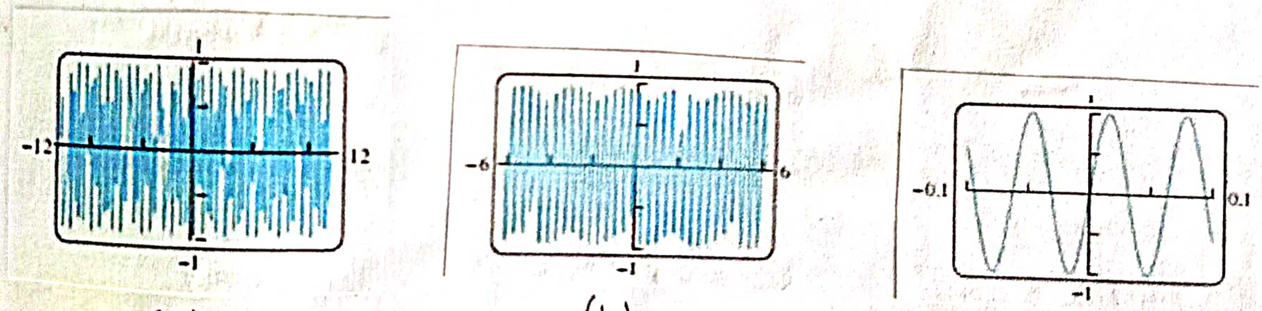


Figure 2.3

2.3 Graphing with Software :

Example : Graph the function $f(x) = \sin 100x$



(a) Viewing window → $[-12, 12]$ by $[-1, 1]$

(b) $[-6, 6]$ by $[-1, 1]$

(c) $[-0.1, 0.1]$ by $[-1, 1]$

Figure 2.4 : Graphs of function $y = \sin 100x$ in three viewing windows. Because the period is $2\pi/100 \approx 0.063$, the smaller window in (c) best displays the true aspects of this rapidly oscillating function.

Chapter Three

3.0 Limits and Continuity

3.1 The limit of a Function

We begin our exploration of limits by taking a look at the graphs of the functions

$$f(x) = \frac{x^2-1}{x-1} \quad , \quad g(x) = \frac{1}{(x-1)^2}$$

which are shown in Figure 3.1. In particular, let's focus our attention on the behavior of each graph at and around $x=1$.

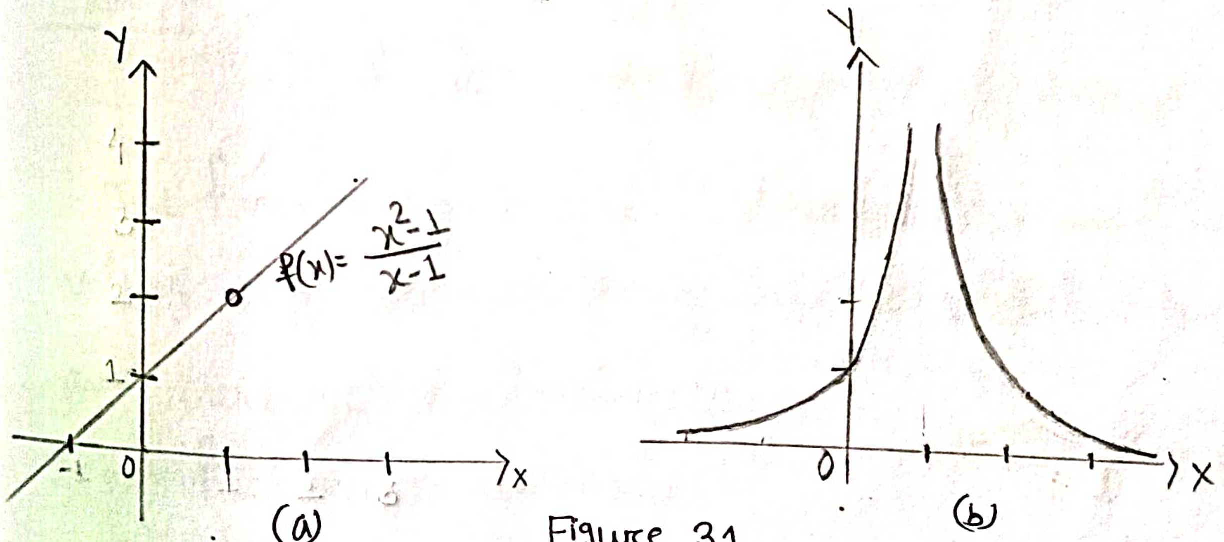


Figure 3.1

Each of the two functions is undefined at $x=1$ but if we make this statement and no other, we give a very incomplete picture of how each function behaves in the vicinity of $x=1$. To express the behaviour of each graph in vicinity of 1 more completely, we need to introduce the concept of limit.

An informal Description of the Limit of a Function :

We now give an informal definition of the limit of a function f at an interior point of the domain of f. Suppose that f(x) is defined on an open interval about c, expect possibly at c itself. If f(x) is arbitrarily close to the number L for all x sufficiently close to c, other than c itself, then we say that f approaches the limit L as x approaches c, and write

$$\lim_{x \to c} f(x) = L$$

Example :

$$g(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

The function g(x) has no limit as $x \to 0$ because the values of g grow arbitrarily lauge in absalute value as $x \to 0$ and therefore do not stay close to any fixed real number (Figure 3.2). We say the function is not bounded.

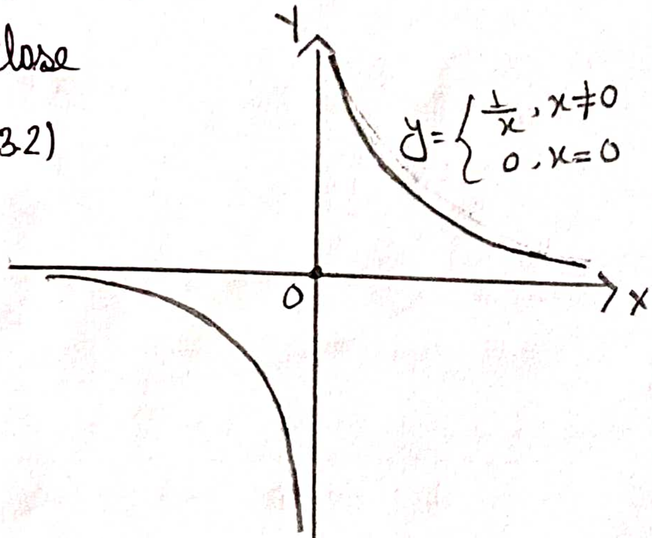
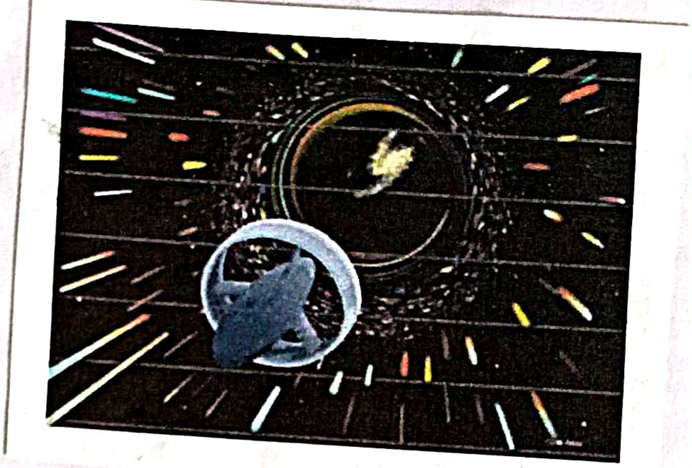


Figure 3.2

Einstein's Equation:

Albert Einstein show that a limit exists to how fast any object can travel.



Given Einstein's equation for the mass of a moving object, what is the value of this bound?

Solution: Our starting point is Einstein's equation for the mass of a moving object, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, where m_0 is the objects mass at rest, v is its speed and c is the speed of light. To see how the mass changes at high speeds, we can graph the ratio of masses m/m_0 as a function of the ratio of speeds, v/c (Figure 3.3).

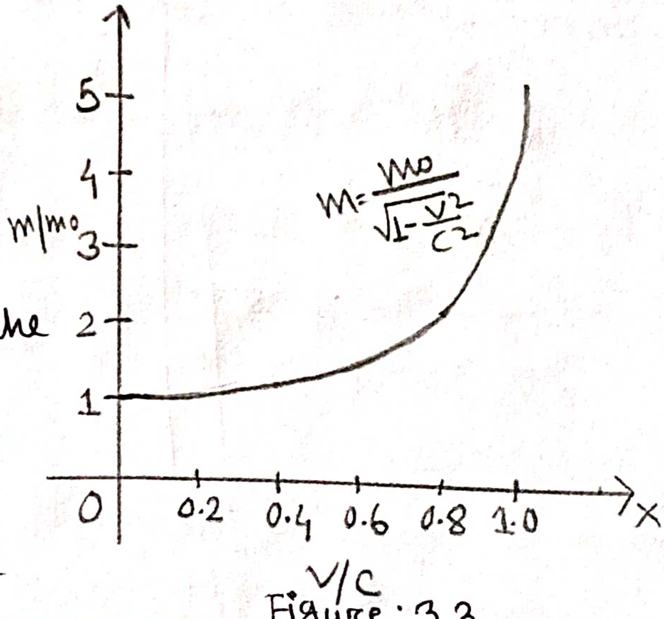


Figure: 3.3

We can see that as the ratio of speeds approaches 1 – that is, as the speed of the object approaches the speed of light - the ratio of masses increases without bound. In other words, the function has a vertical asymptote at $v/c = 1$. We can try a few values of this ratio to test this idea.

$\frac{v}{c}$	$\sqrt{1-\frac{v^2}{c^2}}$	$\frac{m}{m_0}$
0.99	0.1411	7.089
0.999	0.0447	22.37
0.9999	0.0141	70.71

Table : Ratio of Masses and speeds for a Moving object.

Thus according to Table, if an object with mass 100 kg is traveling at 0.9999c, it masses becomes 7071 kg. Since no object can have an infinite mass, we conclude that no object can travel at or more than the speed of light.

3.2 The Precise Definition of a Limit:

Definition: Let $f(x)$ be defined on an open interval about c, except possibly at c itself. We say that the limit of $f(x)$ as x approaches c is the number L, and write $\lim_{x \to c} f(x) = L$

if, for every number $\varepsilon > 0$, $\exists$ a corresponding number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \text{ whenever } 0 < |x - c| < \delta.$$

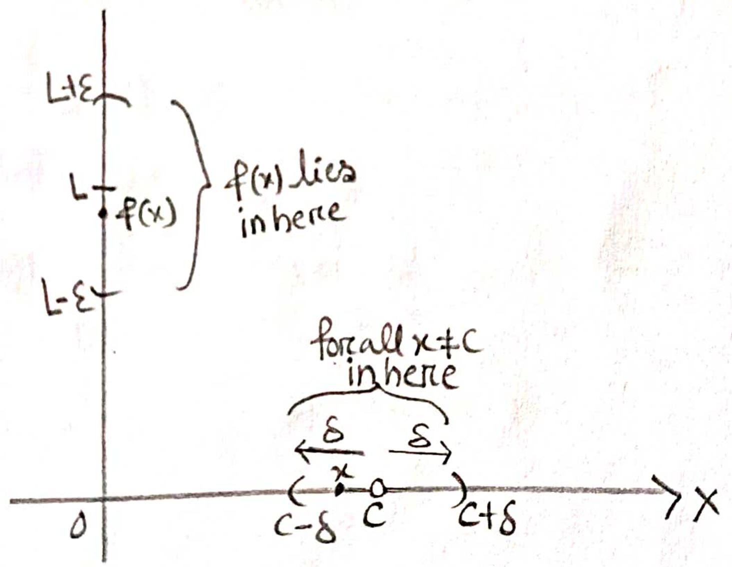


Figure 3.4: The relation of δ and ε in the definition of limit.

Example: Show that

$$\lim_{x \to 1} (5x-3) = 2$$

Solution: Set $c = 1$, $f(x) = 5x-3$ and $L = 2$ in the definition of limit. For any given $\varepsilon > 0$, we have to find a suitable $\delta > 0$ so that if $x \neq 1$ and x is within distance δ of $c = 1$ that is, whenever

$$0 < |x-1| < \delta,$$

it is true that is within distance ε of $L = 2$ so

$$|f(x)-2| < \varepsilon$$

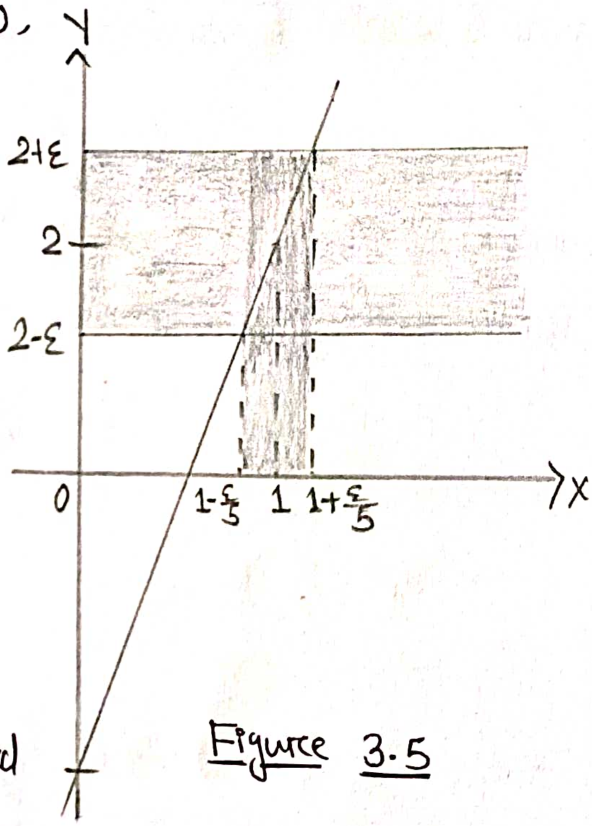


Figure 3.5

Now, find δ by working backward from the ε-inequality

$$|(5x-3)-2| = |5x-5| = 5|x-1| < \varepsilon$$
$$\Rightarrow |x-1| < \varepsilon/5$$

Thus we can take $\delta = \varepsilon/5$ (Figure 3.5). If $0 < |x-1| < \delta = \varepsilon/5$, then

$$|(5x-3)-2| = |5x-5| = 5|x-1| < 5(\varepsilon/5) = \varepsilon$$

which proves that $\lim_{x \to 1} (5x-3) = 2$.

(The value of $\delta = \varepsilon/5$ is not the only value that will make $0 < |x-1| < \delta$ imply $|5x-5| < \varepsilon$).

3.3 Continuity:

Many functions have the property that their graphs can be traced with a pencil without lifting the pencil from the page, such function are called continuous. Other function have points at which a break in the graph occurs, but satisfy this property over intervals contained in their domains. They are continuous on these intervals and are said to have a discontinuity at a point where a break occurs.

Continuity at a point:

Definitions: Let c be a real number that is either an interior point or an endpoint of an interval in the domain of f.

- The function f is continuous at c if

$$\lim_{x \to c} f(x) = f(c)$$

- The function f is right continuous at c if

$$\lim_{x \to c^+} f(x) = f(c)$$

- The function f is left continuous at c if

$$\lim_{x \to c^-} f(x) = f(c)$$

Theorem: A function f defined on an interval I is continuous at a point $c \in I$ iff for every sequence $\langle c_n \rangle$ in I converging to c, we have

$$\lim_{n\to\infty} f(c_n) = f(c)$$

Discontinuous function:

A function is said to be discontinuous at a point c of its domain if it is not continuous there at c. The point c is then called a point of discontinuity of the function.

Example. The Dirichlet's function f defined on $\mathbb{R}$ by

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

is discontinuous at every point.

Theorem: If a function is continuous in a closed interval then it is bounded therein.

—

Chapter Four

4.0 Differentiation

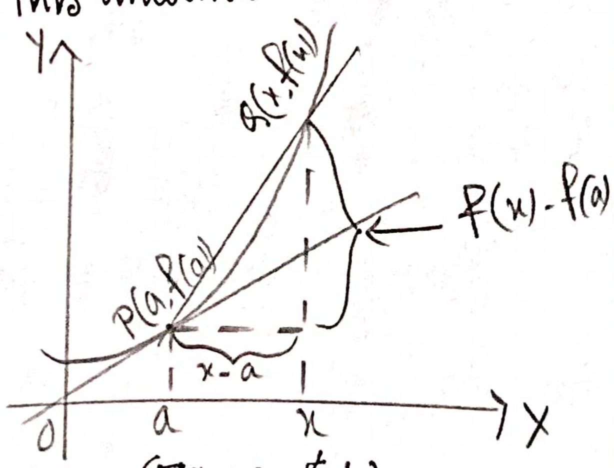
4.1 Tangent Lines and the differentiation at a point:

Finding a tangent line to the Graph of a Function:

If a curve C has equation $y = f(x)$ and we want to find the tangent line to C at the point $P(a, f(a))$, then we consider a near by point $Q(x, f(x))$, where $x \neq a$, and compute the slope of the secant line PQ:

$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$

Then we let Q approach P along the curve C by letting x approach a. If m_{PQ} approaches a number m, then we define the tangent t to be the line through P with slope m. (This amounts to saying that the tangent line is the limiting position of the secant line PQ as Q approaches P.



(Figure- 4.1)

Definition: The tangent line to the curve $y=f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

provided the limit exists.

Rate of Change: Derivatives at a point:

Definition: The derivative of a function f at a point a, denoted by $f'(a)$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

provided the limit exists.

Example: Find the derivative of the function $f(x)=x^2-8x+9$ at the point a.

Solution: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{[(a+h)^2-8(a+h)+9]-[a^2-8a+9]}{h}$

$$= \lim_{h\to 0} \frac{2ah+h^2-8h}{h}$$

$$= \lim_{h\to 0} (2a+h-8)$$

$$= 2a-8$$

4.2 The Derivative as a Function :

In the last section we considered the derivative of a function f at a fixed number 'a'

$$\boxed{1} \quad f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Here we change our point of view and let the number 'a' vary. If we replace 'a' in Equation 1 by a variable x , we obtain

$$\boxed{2} \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Given any number x for which this limit exists, we assign to x the number f'(x). So we can regard f' as a new function, Called the derivative of f and defined by Equation 2.

A function $f(x)$ is said to be differentiable at a if $f'(a)$ exist. More generally, a function is said to be differentiable on S if it is differentiable at every point in an open set S, and a differentiable function is one in which $f'(x)$ exists on its domain.

<u>Example</u>: If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'.

<u>Solution</u>:

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

We see that $f'(x)$ exists if $x > 0$, so the domain of $f'(x)$ is $(0, \infty)$.

Graphing the Derivative:

We have already discussed how to graph a function, so given the equation of a function or the equation of a derivative function, we could graph it. Given both, we would expect to see a correspondence between the graphs of these two functions, since $f'(x)$ gives the rate of change of a function $f(x)$ (or slope of the tangent line to $f(x)$).

In Example we found that for $f(x) = \sqrt{x}$, $f'(x) = 1/2\sqrt{x}$. If we graph these functions on the same axes, as in Figure , we can use the graph to understand the relationship between these two functions. First, we notice that $f(x)$ is increasing over its entire domain, which means that the slopes of its tangent lines at all points are positive. Consequently, we expect $f'(x) > 0$ for all values of x in its domain. Furthermore, as x increases, the slopes of the tangent lines to $f(x)$ are decreasing and we expect to see a corresponding decrease in $f'(x)$. We also observe that $f(0)$ is undefined and that $\lim_{x\to 0^+} f'(x) = +\infty$, corresponding to a vertical tangent to $f(x)$ at 0.

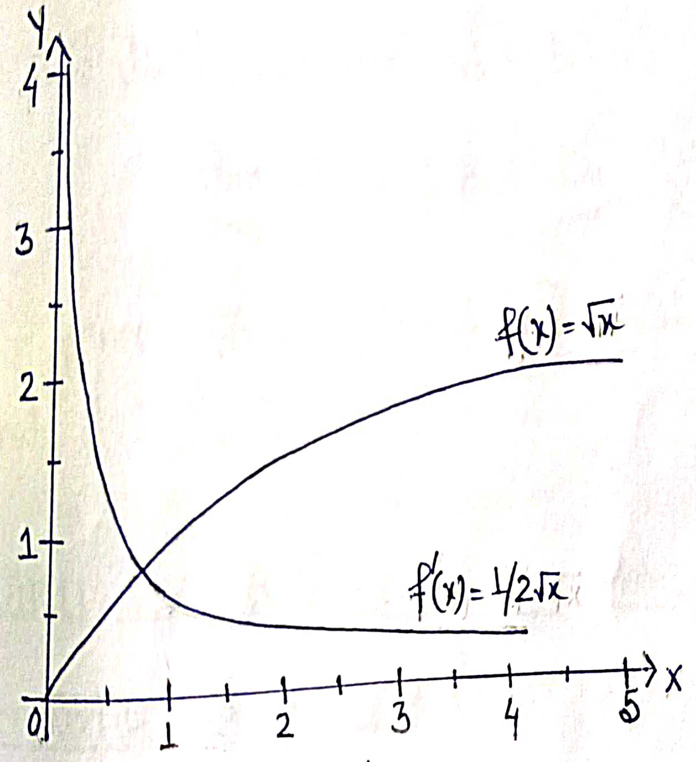


Figure 4.2

Differentiable Functions are continuous:

Theorem: Differentiability implies continuity.
If f has a derivative at $x=c$, then f is continuous at $x=c$.

The converse of the theorem is false. A function need not have a derivative at a point where it is continuous.

For example, $f(x) = |x|$

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{|x|}{x}$$

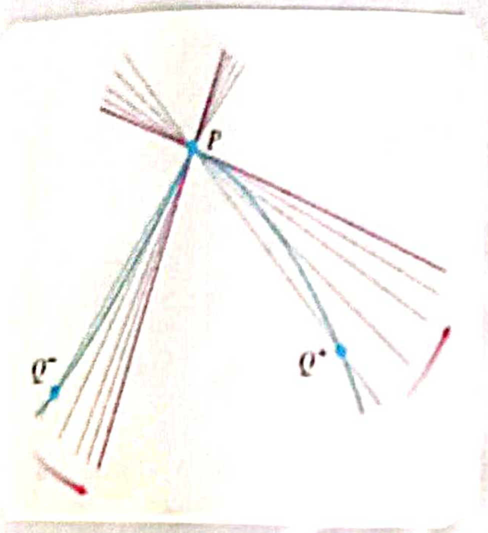
This limit does not exist because

$$\lim_{x \to 0^-} \frac{|x|}{x} = -1 \text{ and } \lim_{x \to 0^+} \frac{|x|}{x} = 1$$

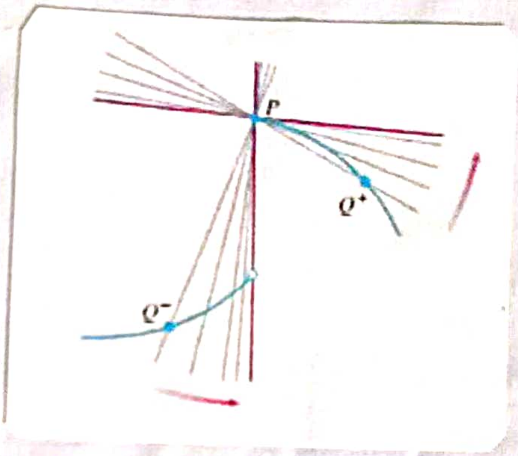
Hence $f(x) = |x|$ is not differentiable at $x = 0$ but it is continuous at $x = 0$.

When Does a Function Not Have a Derivative at a Point?

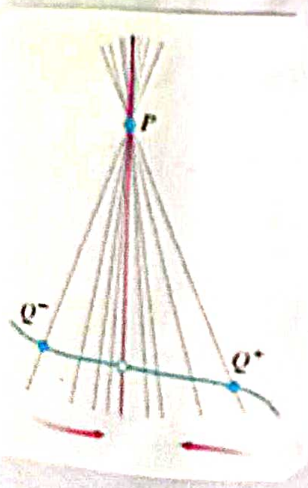
A function has a derivative at a point x_0 if the slope of the secant lines through $P(x_0, f(x_0))$ and a nearby point Q on the graph approach a finite limit as Q approaches P. The differentiability is a "smoothness" condition on the graph of f. A function can fail to have a derivative at a point for many reasons, including the existance of points where the graph has



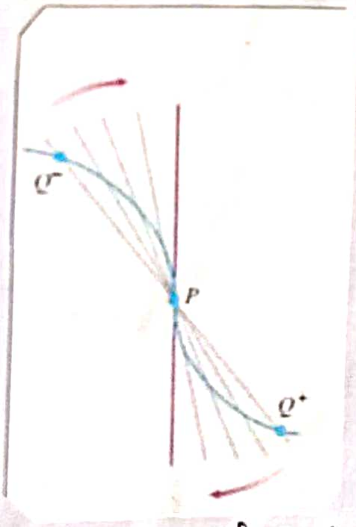
1) a corner, where one sided derivatives differ



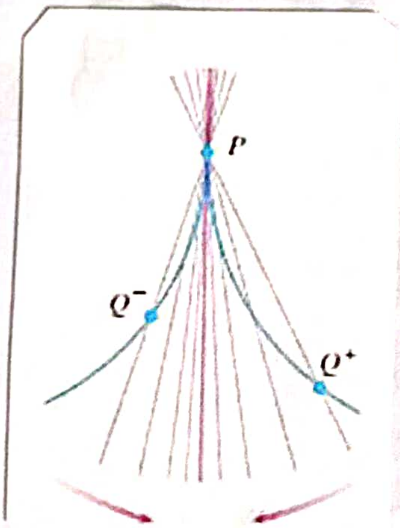
2) a discontinuity.



3) a discontinuity



4) a vertical tangent line, where the slope of PQ approaches ∞ from both sides or approaches −∞ from both sides.



5) a cusp, where the solpe of PQ approaches ∞ from one side and −∞ from the other.

4.3 Rules of Differentiation:

In this section we learn how to differentiate constant functions, power functions, polynomials, rational functions and certain combinations of them, simply and directly, without having to take limits each time.

Derivative of a Constant Function:

If f has the constant value $f(x) = c$, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0$$

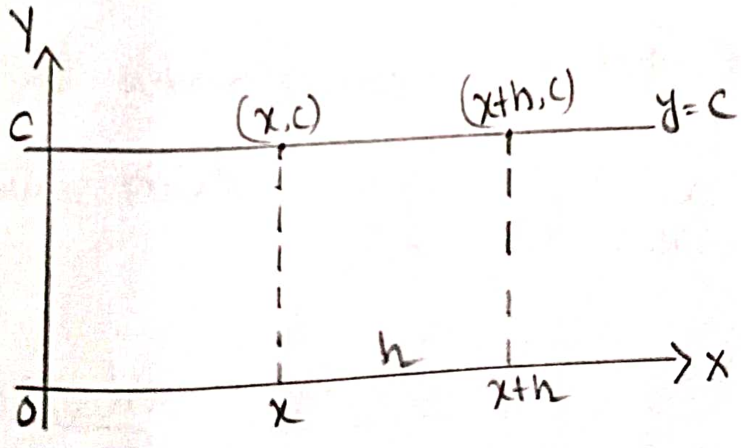


Figure 4.3: The rule $\frac{d}{dx}(c) = 0$ is another way to say that the values of constant functions never change and that the slope of a horizontal line is zero at every point.

Derivative of a Positive integer power:

If n is a positive integer, then

$$\frac{d}{dx} x^n = nx^{n-1}$$

Generally,

If n is any real number, then

$$\frac{d}{dx} x^n = nx^{n-1}$$

for all x where the powers x^n and x^{n-1} are defined.

Derivative Sum Rule:

If u and v are differentiable functions of x, then their sum $u+v$ is differentiable at every point where u and v are both differentiable. At such points

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

Derivative product Rule:

If u and v are differentiable at x, then so is their product uv and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

CHAPTER FIVE

5.0 Application of Differentiation

One of the most important applications of the derivative is its use as a tool for finding the optimal solutions to problems. optimization problem abound in mathematics, physical science and engineering, business and economics and biology and medicine.

In this chapter we apply differentiation to solve applied optimization problems, such as maximizing revenue and minimizing surface area etc.

5.1 Related Rates :

In this section we look at questions that arise when two or more related quantities are changing. The problem of determining how the rate of change of one of them affects the rate of change of the others is called a related rates problem.

Setting up Related - Rates Problems :

In many real world applications, related quantities are changing with respect to time. For example, Suppose we are pumping air into a spherical balloon. Both the volume and radius of the balloon are increasing over time. If V is the volume and r is the radius of the ballon at an instant time, then

$$V = \frac{4}{3}\pi r^3$$

Here we can say that rate of change in the volume, V is related to the rate of change in the radius r. In this case, we say that $\frac{dV}{dt}$ and $\frac{dr}{dt}$ are related rates because V is related to r. Here we study several examples of related quantities that are changing with respect to time and we look at how to calculate one rate of change given another rate of change.

Example 5-1 :

A spherical balloon is being filled with air at the constant rate of $2 cm^3/Sec$. How fast is the radius increasing when the radius is 3cm ?

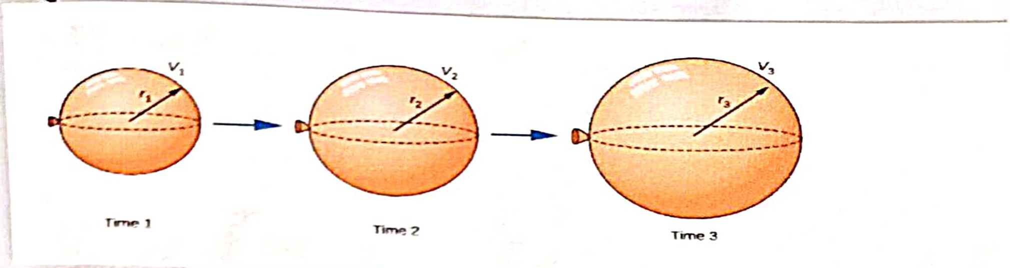


Fig5-1 As the ballon is being filled with air, both the radius and the volume are increasing w.r.t time.

Solution :

The volume of the sphere of radius r is

$$V = \frac{4}{3}\pi r^3 \text{ cm}^3$$

Since the balloon is being filled with air, both the volume and the radius are function of time.

Therefore, t seconds after beginning to fill the balloon with air, the volume of air in the balloon is

$$V(t) = \frac{4}{3}\pi [r(t)]^3 \text{ cm}^3$$

Differentiating both sides of this equation with respect to time and applying the chain rule, we see that the rate of change in the volume is related to the rate of change in the radius by the equation

$$V'(t) = 4\pi\left[r(t)\right]^2 r'(t)$$

The balloon is being filled with air at the constant rate of $2\,cm^3/sec$, so $V'(t) = 2\,cm^3/sec$.

Therefore $2\,cm^3/sec = \left(4\pi\left[r(t)\right]^2 cm^2\right)\left(r'(t)\,cm/s\right)$

Which implies $r'(t) = \dfrac{1}{2\pi(r(t))^2}\,cm/sec.$

When the radius $r = 3$ cm

$$r'(t) = \frac{1}{18\pi}\,cm/sec.$$

Example 5.2: A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

Solution :

Let x be the distance from the man to the point on the path closest to the searchlight. We let θ be the angle between the beam of the searchlight and the perpendicular to the path.

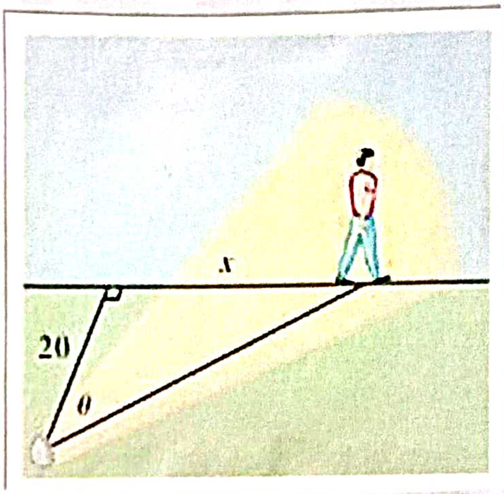


Figure 5.2

We are given that $dx/dt = 4\,ft/s$ and are asked to find $d\theta/dt$ when $x = 15$. The equation that relates x and θ can be written from the figure.

$$\frac{x}{20} = \tan\theta \Rightarrow x = 20\tan\theta$$

$$\Rightarrow dx/dt = 20\sec^2\theta \, d\theta/dt$$

So $d\theta/dt = \frac{1}{20}\cos^2\theta \frac{dx}{dt} = \frac{1}{20}\cos^2\theta\,(4) = \frac{1}{5}\cos^2\theta$

When $x = 15$, the length of the beam is 25,

so $\cos\theta = \frac{4}{5}$ and $\frac{d\theta}{dt} = \frac{1}{5}\left(\frac{4}{5}\right)^2 = \frac{16}{25} = 0.128$

$\therefore$ The searchlight is rotating at rate of $0.128\,rad/s$.

5.2 Applied Optimization Problems:

One common application of calculus is calculating the maximum or minimum value of a function. For example, companies often wants to minimize production costs or maximize revenue. In this section we use differentiation to solve a variety of optimization problems in mathematics, physics, economics and business.

Solving Optimization Problems:

Example (Maximizing Area): A hobby store has 20ft of fencing to fence off a rectangular area for an electric train in one corner of its display room. The two sides up against the wall require no fence. What dimensions of the rectangle will maximize the area? What is the maximum area?

Solution: Let's first make a drawing and express the area in one variable.

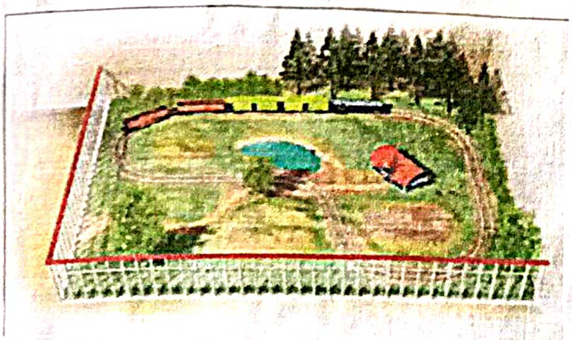


Figure 5.3

If we let x = the length in feet of one side, y = the length in feet of the other. Then since the sum of the lengths must be 20 ft, we have

$$x+y=20 \text{ and } y=20-x$$

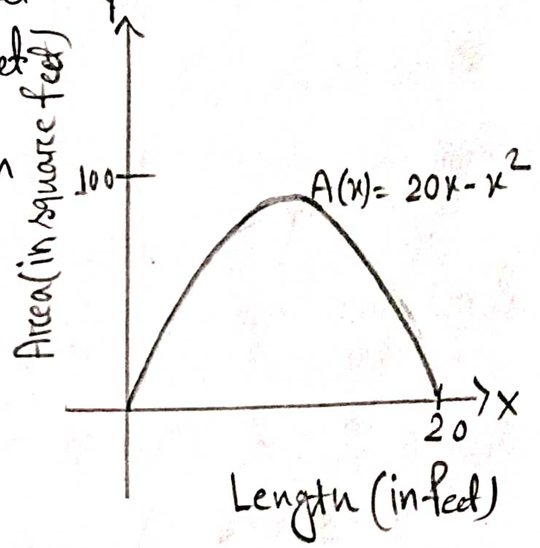


Figure – 5.4

Thus the area is given by

$$A=xy=x(20-x)=20x-x^2$$

We are trying to find the maximum value of

$A(x)=20x-x^2$, over the interval $(0,20)$. We consider the interval $(0,20)$ because x is a length and cannot be negative or 0. Since there is only 20 ft of fencing, x cannot be greater than 20. Also, x cannot be 20 because then the length of y would be 0.

a) We first find $A'(x)$: $A'(x)=20-2x$.

b) This derivative exists for all values of x in $(0,20)$. Thus the only critical values are where $A'(x)=20-2x=0 \Rightarrow x=10$

Since, there is only one critical value, we can use the 2nd derivative to determine whether we have maximum.

$A''(x)=-2<0$, so $A''(10)$ also negative and $A(10)$ is maximum.

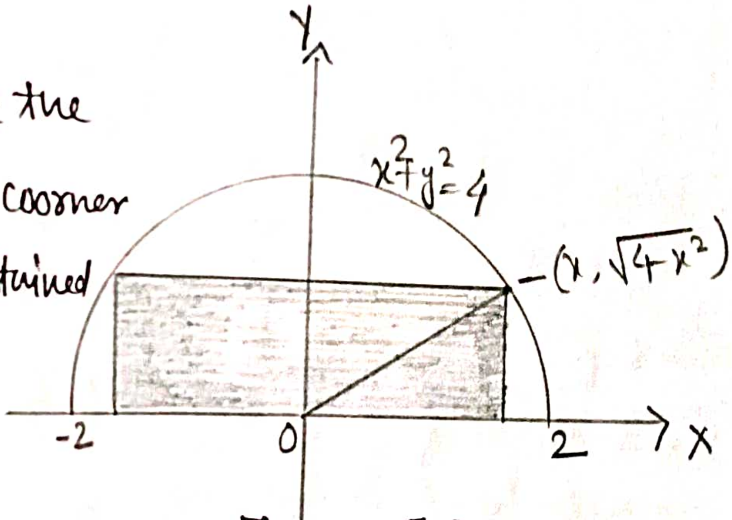
Now, $A(10)=10(20-10)=100$

Thus the maximum area of 100 ft^2 is obtained using 10 ft for the length of one side and 20-10, or 10 ft for the other.

<u>Example</u>: A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area the rectangle can have and what are its dimensions?

<u>Solution</u>:

Let $(x, \sqrt{4-x^2})$ be the coordinates of the corner of the rectangle obtained by placing the circle and rectangle in the coordinate plane (Figure 5.).



<u>Figure - 5.5</u>

The length, height and area of the rectangle can then be expressed in terms of the position x of the lower right hand corner.

Length: $2x$, Height: $\sqrt{4-x^2}$, Area: $2x\sqrt{4-x^2}$

The values of x are to be found in the interval $0 \leq x \leq 2$, where the selected corner of the rectangle lies.

we have to find the absolute maximum value of the function

$$A(x) = 2x\sqrt{4-x^2} \quad \text{on the domain } [0,2]$$

The derivative

$$\frac{dA}{dx} = \frac{-2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2}$$

is not defined when $x=2$ and is equal to zero when

$$\frac{-2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2} = 0$$

$$\Rightarrow -2x^2 + 2(4-x^2) = 0$$

$$\Rightarrow x = \pm\sqrt{2}$$

of the two zeros $x=\sqrt{2}$ and $x=-\sqrt{2}$, only $x=\sqrt{2}$ lies in the interior of A's domain and makes the critical point list. The values of A at the endpoints and at this one critical point are

Critical point value : $A(\sqrt{2}) = 2\sqrt{2}\sqrt{4-2} = 4$

Endpoint values : $A(0) = 0$, $A(2) = 0$

The area has a maximum value of 4 when the rectangle is $\sqrt{4-x^2} = \sqrt{2}$ unit high and $2x = 2\sqrt{2}$ units long.

5.3 Maxima and Minima

Given a particular function, we are often interested in determining the largest and smallest values of the function. This information is important in creating accurate graphs. Finding the maximum and minimum values of a function also has practical significance because we can use this method to solve optimization problem, such as maximizing profit, finding the maximum height a rocket can reach. In this section, we look at how to use differentiation to find the largest and smallest values for a function.

Absolute Extrema:

Consider the function $f(x) = x^2+1$ over the interval $(-\infty, \infty)$. As $x \to \pm\infty$, $f(x) \to \infty$. Therefore the function does not have a largest value. However, since $x^2+1 \geq 1$ for all real number x and $x^2+1 = 1$ when $x = 0$, the function has a smallest value 1, when $x = 0$. We say that 1 is the absolute minimum of $f(x) = x^2+1$. And $f(x) = x^2+1$ does not have an absolute maximum (Fig- 5.6).

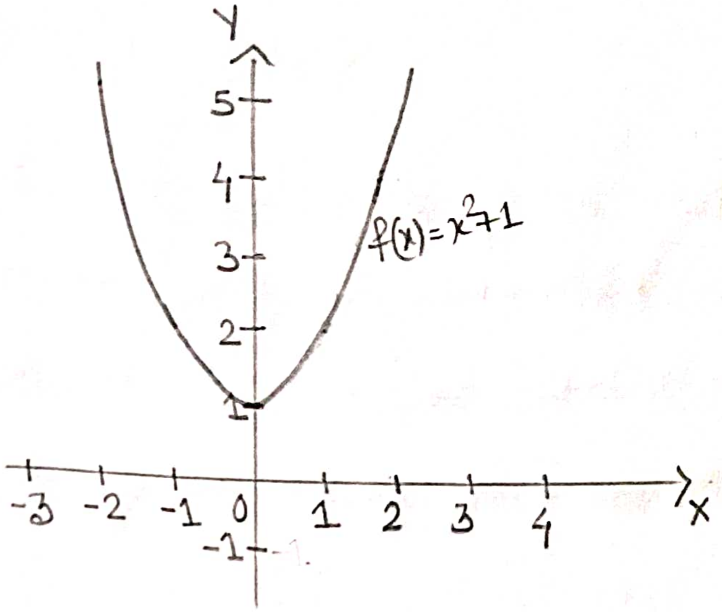


Fig 5.6 - The given function has an absolute minimum at $x=0$. The function does not have an absolute maximum.

DEFINITIONS:

Let f be a function with domain D. Then f has an absolute maximum value on D at a point c if

$$f(x) \leq f(c), \text{ for all } x \text{ in } D.$$

and an absolute minimum value on D at c if

$$f(x) \geq f(c), \text{ for all } x \text{ in } D.$$

Maximum and minimum values are called extreme values of the function f. Absolute maxima or minima are also referred to as global maxima or minima.

5.4 The Mean Value Theorem:

The Mean value Theorem is one of the most important theorems in calculus. We look at some of its applications. First, let's start with a special case of the Mean value Theorem, called Rolle's Theorem.

Ralle's Theorem:

Suppose that $y = f(x)$ is continuous over the closed interval $[a,b]$ and differentiable at every point of its interior (a,b). If $f(a) = f(b)$, then there is at least one number c in (a,b) at which $f'(c) = 0$.

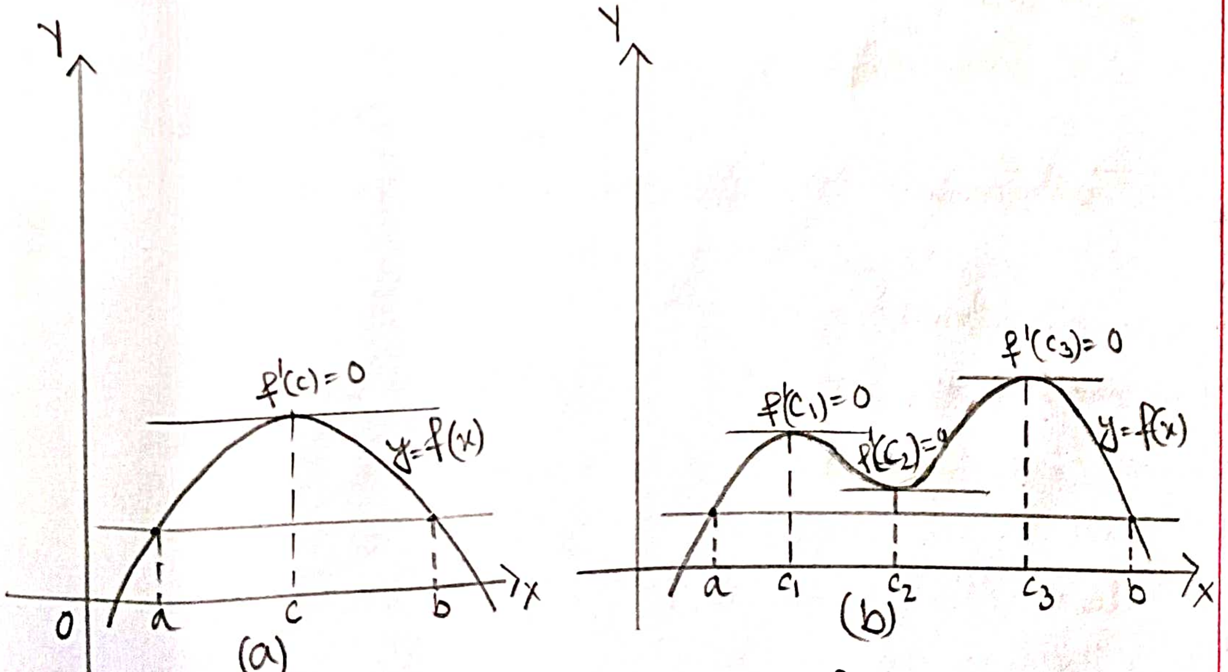


Figure 57 - Ralle's Theorem says that a differentiable curve has at least one horizontal tangent between any two points where it crosses a horizontal line.

The Mean Value Theorem:

The Mean value theorem is a slanted version of Rolle's Theorem (Figure 5.8). The Mean value Theorem guarantees that there is a point where the tangent line is parallel to the secant line that joins A and B.

Theorem (The Mean Value Theorem):

Suppose $y = f(x)$ is continuous over a closed interval $[a, b]$ and differentiable on the interval (a, b). Then there is at least one point c in (a, b) at which

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

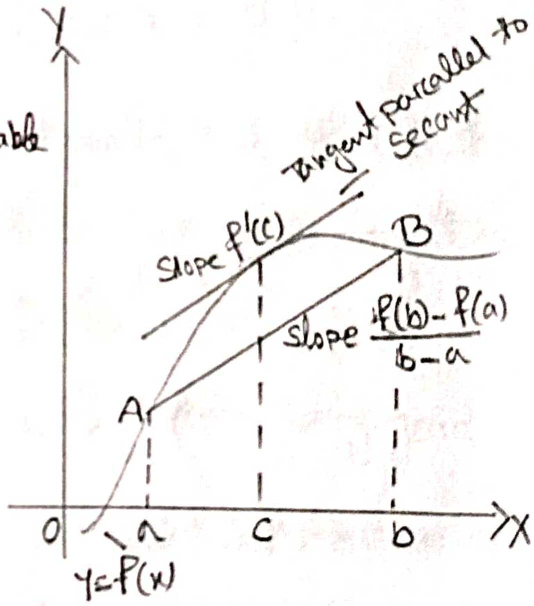


Figure 5.8 – Geometrically, the Mean value theorem says that somewhere between a and b the curve has at least one tangent line parallel to the secant line that joins A and B.

Conclusion :

Calculus has been an important aspect of mathematics that the significance of its study cannot be over emphasized.

This aspect of mathematics is divided into two parts: Differentiation And Integration. However our emphasis is on Differentiation.

Most companies, private organization and factories mobilize their staff to produce one goods or the other with respect to time. So this aspect of Calculus called differentiation serves as a very good method use in maximing production and distribution of raw and finished goods as at when needed.

In conclusion, this aspect of calculus (differentiation) is a needed factor that must be in mind in the course of any production.

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