Total number of printed pages–8

3 (Sem–5/CBCS) MAT HC 1

2021

(Held in 2022)

MATHEMATICS

(Honours)

Paper : MAT–HC–5016

(Riemann Integration and Metric Spaces)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×10=10

   (a) Describe an open ball in the discrete metric space.

   (b) Find the derived set of the sets $(0,1]$ and $[0,1]$.

   (c) A subset B of a metric space (X,d) is open if and only if

   (i) $B = \overline{B}$

   (ii) $B = B^{o}$

   (iii) $B \neq \overline{B}$

   (iv) $B \neq B^{o}$

   (Choose the correct one)

Contd.

(d) Which of the following is false ?

(i) $\phi^o = \phi$, $X^o = X$

(ii) $A \subseteq B \Rightarrow A^o \subseteq B^o$

(iii) $(A \cap B)^o = A^o \cap B^o$

(iv) $(A \cup B)^o = A^o \cup B^o$

where A, B are subsets of a metric space (X, d). *(Choose the false one)*

(e) The closure of the subset $F = \left\{1, \frac{1}{2}, \frac{1}{3}, ...\right\}$ of the real line $\mathbb{R}$ is

(i) ϕ

(ii) F

(iii) $F \cup \{0\}$

(iv) $F - \{0\}$

(Choose the correct one)

(f) In a metric space an arbitrary union of closed sets need not be closed. Justify it with an example.

(g) If A is a subset of a metric space (X, d), then which one is true ?

(i) $d(A) = d(\overline{A})$

(ii) $d(A) \neq d(\overline{A})$

(iii) $d(A) > d(\overline{A})$

(iv) $d(A) < d(\overline{A})$

(Choose the true one)

(h) When is an improper Riemann integral said to be convergent ?

(i) Evaluate $\int_0^\infty e^{-x} dx$ if it exists

(j) Show that $\Gamma(1) = 1$

2. Answer the following questions : 2×5=10

(a) Let F be a subset of a metric space (X, d). Prove that the set of limit points of F is a closed subset of (X, d).

(b) If F_1 and F_2 are two subsets of a metric space (X, d), then $\overline{F_1 \cap F_2} = \overline{F_1} \cap \overline{F_2}$. Justify whether it is false *or* true.

(c) Let (X, d_X) and (Y, d_Y) be metric spaces and $f: X \to Y$. If for all subsets A of X, $f(\overline{A}) \subseteq \overline{f(A)}$, then show that f is continuous on X.

(d) Let $f:[a, b] \to \mathbb{R}$ be integrable. Show that $|f|$ is integrable.

(e) Show that the function $f:[a, b] \to \mathbb{R}$ defined by $f(x) = c$ for all $x \in [a, b]$ is integrable with its integral $c(b-a)$.

3. Answer ***any four*** parts : 5×4=20

(a) Define a complete metric space. Show that the metric space $X = \mathbb{R}^n$ with the metric given by

$$d_p(x, y) = \left(\sum |x_i - y_i|^p\right)^{\frac{1}{p}}, \ p \geq 1$$

where $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ are in $\mathbb{R}^n$, is a complete metric space. 1+4=5

(b) Let (X, d_X) and (Y, d_Y) be metric spaces. Prove that a mapping $f: X \to Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y. 5

(c) Prove that if the metric space (X, d) is disconnected, then there exists a continuous mapping of (X, d) onto the discrete two-element space (X_0, d_0). 5

(d) Let $f:[a, b] \to \mathbb{R}$ be a continuous function. Prove that f is integrable. 5

(e) Discuss the convergence of the integral $\int_1^\infty \frac{1}{x^p}\, dx$ for various values of p. 5

(f) Show that for $a > -1$,

$$S_n = \frac{1^n + 2^n + \dots\dots + n^n}{n^{1+a}} \to \frac{1}{1+a}.$$ 5

4. Answer ***any four*** parts : 10×4=40

(a) (i) Let (X, d) be a metric space. Define $d : X \times X \to \mathbb{R}$ by $d'(x, y) = \dfrac{d(x, y)}{1+d(x, y)}$ for all $x, y \in X$. Prove that d' is a metric on X.

Also show that d and d' are equivalent metrices on X. 4+2=6

(ii) Prove that a convergent sequence in a metric space is a Cauchy sequence. 4

(b) (i) Let (X, d) be a metric space and F be a subset of X. Prove that F is closed in X if and only if F^c is open. 5

(ii) If (Y, d_Y) is a subspace of a metric space (X, d), then show that a subset Z of Y is open in Y if and only if there exists an open set $G \subseteq X$ such that $Z = G \cap Y$. 5

(c) Prove that a metric space (X, d) is complete if and only if for every nested sequence $\{F_n\}_{n \geq 1}$ of non-empty closed subsets of X such that $d(F_n) \to 0$ as $n \to \infty$, the intersection $\bigcap_{n=1}^{\infty} F_n$ contains one and only one point. 10

(d) (i) Prove that in a metric space (X, d), each open ball is an open set. 4

(ii) Let (X, d_X) and (Y, d_Y) be metric spaces and $A \subseteq X$. Prove that a function $f : A \to Y$ is continuous at $a \in A$ if and only if whenever a sequence $\{x_n\}$ in A converges to a, the sequence $\{f(x_n)\}$ converges to $f(a)$. 6

(e) (i) Define uniformly continuous mapping in a metric space. Give an example to show that a continuous mapping need not be uniformly continuous. 1+4=5

(ii) Prove that the image of a Cauchy sequence under a uniformly continuous mapping is itself a Cauchy sequence. 5

(f) Let $(\mathbb{R}, d)$ be the space of real numbers with the usual metric. Prove that a subset $I \subseteq \mathbb{R}$ is connected if and only if I is an interval. 10

(g) Let $f:[a, b] \rightarrow \mathbb{R}$ be a bounded function. Show that f is integrable if and only if it is Riemann integrable. 10

(h) (i) State and prove first fundamental theorem of calculus. Using it show that

$$\int_0^a f(x)dx = \frac{a^4}{4} \text{ for } f(x) = x^3.$$

1+3+2=6

(ii) Let f be continuous on $[a, b]$. Prove that there exists $c \in [a, b]$ such that $\frac{1}{b-a}\int_a^b f(x)dx = f(c)$.

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3 (Sem–5 /CBCS) MAT HC 2

2021

(Held in 2022)

MATHEMATICS

(Honours)

Paper : MAT–HC–5026

(Linear Algebra)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×10=10

   (i) Is $\mathbb{R}^2(\mathbb{R})$ is a subspace of $\mathbb{R}^3(\mathbb{R})$?

   (ii) Let A be a 5×4 matrix. If null space of A is a subspace of $\mathbb{R}^k$ then what is k?

   (iii) Let S be a subset of a vector space $V(F)$ and S contains zero vector of V. Then S is

   (A) linearly independent

   (B) linearly dependent

(C) Both linearly independent and linearly dependent

(D) None of the above

(Choose the correct option)

(iv) Write the standard basis of the vector space of polynomial in x with real co-efficient of degree ≤ 3.

(v) "The eigenvalues of a triangular matrix are the entries on its main diagonal." *(State True **or** False)*

(vi) Define inner product on $\mathbb{R}^n$.

(vii) Which vector is orthogonal to every vector in $\mathbb{R}^n$?

(viii) How do you explain $dim\ W = 1$ geometrically where W is a subspace of the vector space $\mathbb{R}^3(\mathbb{R})$?

(ix) Let A be the 4×4 real matrix,

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -2 & -2 & 0 \\ 1 & 1 & -2 & 1 \end{bmatrix}$$

Then the characteristic polynomial for A is

(A) $x^2(x-1)^2$

(B) $(x-1)^2(x+1)^2$

(C) $x^2(x+1)^2$

(D) None of the above

(Choose the correct option)

(x) What do you mean by the length of a vector in $\mathbb{R}^n$?

2. Answer the following questions : $2 \times 5 = 10$

(i) Let V be the vector space of all functions from the real field $\mathbb{R}$ to $\mathbb{R}$. Show that $W = \{f : f(7) = 2 + f(1)\}$ is not a subspace of V.

(ii) Show that every subset of an independent set is independent.

(iii) Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Is v a eigenvector of A ?

(iv) Let T be the linear operator on $\mathbb{R}^3$ defined by $T(a,b,c) = (a+b,\, b+c,\, 0)$. Show that the xy-plane $= \{(x,y,0) : x, y \in \mathbb{R}\}$ is T-invariant subspace of $\mathbb{R}^3$.

(v) Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y onto u.

3. Answer ***any four*** questions : 5×4=20

(i) Prove that the non-zero vectors $v_1, v_2, \ldots, v_n$ are linearly dependent if and only if one of them is a linear combination of the preceding vectors.

(ii) Let $v_1, v_2, \ldots, v_n$ be non-zero eigenvectors of an operator $T : V \to V$ corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Prove that $v_1, v_2, \ldots, v_n$ are linearly independent.

(iii) Let A and B be two similar matrices of order $n \times n$. Prove that A and B have same characteristic polynomial and hence the same eigenvalues.

(iv) Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix}$. An eigenvalue of A is 2. Find a basis for the corresponding eigenspace.

(v) Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^2, given that $A = PDP^{-1}$ where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$.

(vi) Define orthogonal set. If $S = \{u_1, u_2, \ldots, u_p\}$ is an orthogonal set of non-zero vectors in $\mathbb{R}^n$, then prove that S is linearly independent and hence is a basis for the subspace spanned by S.

4. *(i)* If a vector space V has a basis $B = \{v_1, v_2, \ldots, v_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent. Also show that every basis of V must consist of exactly n vectors. 5+5=10

OR

Let U and V be vector spaces over the same field. Let $\{u_1, u_2, \ldots, u_n\}$ be a basis of U and let $v_1, v_2, \ldots, v_n$ be any arbitrary vectors in V. Prove that there exists a unique linear mapping $f : U \to V$ such that

$f(u_1) = v_1,\ f(u_2) = v_2, \ldots, f(u_n) = v_n$ 10

(ii) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. 10

OR

State Cayley-Hamilton theorem for matrices. Use it to express $2A^5 - 3A^4 - A^2 - 4I$ as a linear polynomial in A, when $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. 10

(iii) Let T be the linear operator on $\mathbb{R}^3$, defined by

$T(x, y, z) = (2y + z,\ x - 4y,\ 3x)$

(a) Find the matrix of T in the basis $\{e_1 = (1,1,1),\ e_2 = (1,1,0),\ e_3 = (1,0,0)\}$

(b) Verify that $[T]_e [v]_e = [T(v)]_e$ for any vector $v \in \mathbb{R}^3$. 4+6=10

OR

An $n \times n$ matrix A is diagonalizable if and only if A has n linearly independent eigen-vectors. 10

(iv) Define orthonormal set and orthonormal basis in $\mathbb{R}^n$. Show that $\{u_1, u_2, u_3\}$ is an orthonormal basis of $\mathbb{R}^3$, where

$$u_1 = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix},\quad u_2 = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix},\quad u_3 = \begin{bmatrix} -1/\sqrt{66} \\ -4/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}$$

1+1+8=10

Contd.

OR

Define inner product space. Show that the following is an inner product in $\mathbb{R}^2$:

$$\langle u, v \rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$$

where $u=(x_1, x_2)$, $v=(y_1, y_2)$.

Also show that for all u, v in $\mathbb{R}^2$

$\|u+v\| \leq \|u\| + \|v\|$ 2+5+3=10

Total number of printed pages–7

3 (Sem–5) MAT M 5

2021

(Held in 2022)

MATHEMATICS

(Major)

Paper : 5·5

(Probability)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

   (a) Define probability density function for a continuous random variable.

   (b) What conclusion can be made about the conditional probability $P(A|B)$ if $P(B)=0$?

   (c) State the multiplication theorem of expectation.

Contd.

(d) Write the sample space for the experiment of tossing three coins at a time.

(e) Under what condition $\mathrm{Cov}(X, Y) = 0$?

(f) Choose the correct option for binomial distribution

(i) variance = mean

(ii) variance > mean

(iii) variance < mean

(g) If the mean, median and mode of a continuous distribution coincide, name the distribution.

(h) For a Bernoulli random variable X with $P(X=0)=1-p$ and $P(X=1)=p$, write $E(X)$ and $V(X)$ in terms of p.

2. Answer the following questions : $3\times4=12$

(a) If X and Y are two random variables, show that
$\mathrm{Cov}(X, Y) = E(XY) - E(X)E(Y)$

(b) Find k, such that the function f defined by

$$f(x) = \begin{cases} kx^2 & , \text{ when } 0 < x < 1 \\ 0 & , \text{ otherwise} \end{cases}$$

is a probability density function. Also determine $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$.

(c) Show that in a frequency distribution (x_i, f_i); $i = 1, 2, \ldots\ldots, n$ mathematical expectation of the random variable is nothing but its arithmetic mean.

(d) Define Poisson distribution and hence prove that $\sum_{r=0}^{\infty} p(r) = 1$.

3. Answer **any two** of the following : $5\times2=10$

(a) If A and B are independent events, then show that $\overline{A}$ and $\overline{B}$ are also independent events.

(b) A bag contains five balls and it is known how many of these are white. Two balls are drawn and are found to be white. What is the probability that all are white?

(c) A coin is tossed until a head appears. What is the expectation of the number of heads required?

4. Answer **any two** of the following : 5×2=10

(a) The probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x+y) = x+y, \ 0 < x+y < 1$$
$$= 0, \text{ elsewhere}$$

Evaluate $P\left(X < \frac{1}{2}, Y > \frac{1}{4}\right)$

(b) X is a discrete random variable having the following probability mass function :

Mass points x	: 0	1	2	3	4	5	6	7
$P(X=x)$	: 0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Determine the constant k.

(ii) Find $P(X<6)$.

(iii) What will be $P(X \geq 6)$?

(c) A random variable has the following probability distribution :

x	: 0	1	2	3
$p(x)$	: 0·1	0·3	0·4	0·2

Find (i) $E(X)$, and (ii) $\operatorname{Var}(X)$.

5. Answer **any two** of the following : 5×2=10

(a) If X be a Poisson distributed random variable with the parameter μ, then prove that $E(X) = \mu$ and $V(X) = \mu$

(b) Prove that the mean and variance of a binomially distributed random variable are respectively $\mu = np$ and $\sigma^2 = npq$ (where the symbols have their usual meanings).

(c) Write the probability density function of a random variable X which follows normal distribution with mean μ and variance σ^2. What is standard normal variate ? Find its mean and variance.

6. Answer **any two** of the following : 5×2=10

(a) A random variable X has the density function given by

$$f(x) = 2e^{-2x}, \quad x \geq 0$$
$$= 0, \quad x < 0$$

Find—

(i) the moment generating function;

(ii) the first four moments about the origin.

(b) Find the moment generating function of a continuous probability distribution, whose density is $\frac{1}{2}x^2 e^{-x}$, $0 < x < \infty$ and deduce the values of mean and variance.

(c) State and prove Bayes' theorem.

(d) Let the events $A_1, A_2, \ldots, A_k$ be independent and $P(A_i) = p_i$. Show that the probability that at least one of these events will occur is $1 - \prod_{i=1}^{k}(1 - p_i)$.

Total number of printed pages–16

3 (Sem–5/CBCS) MAT RE 1/RE 2

2021

(Held in 2022)

MATHEMATICS

(Regular Elective)

OPTION–A

Paper : MAT–RE–5016

(Number Theory)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***either*** *in English* ***or*** *in Assamese.*

PART–A

1. Choose the correct option from the following : $1 \times 10 = 10$

তলৰ প্ৰশ্নকেইটাৰ শুদ্ধ উত্তৰ বাছি উলিওৱা ঃ

(i) If a and b are any two integers, then there exists x and y such that,

যদি a আৰু b দুটা যিকোনো অখণ্ড সংখ্যা হয়, তেতিয়াহলে তাত x আৰু y দুটা সংখ্যা থাকিব, যাতে

(a) $gcd(a, b) = ax + by$

Contd.

(b) $gcd(a, b) = ax - by$

(c) $gcd(a, b) = ax^n + by^n$

(d) $gcd(a, b) = (ax + by)^n$

(ii) Let m be a positive integer. Two integers a and b are congruent modulo m iff

ধৰা হ'ল m এটা ধনাত্মক সংখ্যা। a আৰু b অখণ্ড সংখ্যা দুটা congruent modulo m হব যদি আৰু একমাত্ৰ যদিহে

(a) $m | (a - b)$

(b) $m | (a + b)$

(c) $m | (a \times b)$

(d) Both (b) and (c)
[(b) আৰু (c) উভয়ে]

(iii) If (যদি) $a \equiv b \,(mod\, m)$ and (আৰু) $c \equiv d \,(mod\, m)$, then (তেতিয়াহলে)

(a) $a + c \equiv b + d \,(mod\, m)$

(b) $a - c \equiv b - d \,(mod\, m)$

(c) $a \cdot c \equiv b \cdot d \,(mod\, m)$

(d) All of the above
(ওপৰৰ আটাইবোৰ)

(iv) The RRS modulo 6 contains the set of integers

RRS modulo 6 ত তলৰ অখণ্ড সংখ্যাৰ সংহতিটো হব

(a) $\{0, 5\}$

(b) $\{1, 5\}$

(c) $\{1, 2, 3\}$

(d) $\{1, 3, 6\}$

(v) The solution of the linear congruence $2x \equiv 1 \,(mod\, 3)$ is

$2x \equiv 1 \,(mod\, 3)$ ৰৈখিক congruence টোৰ সমাধান হব

(a) $x \equiv 2 \,(mod\, 3)$

(b) $x \equiv 1 \,(mod\, 3)$

(c) $x \equiv 0 \,(mod\, 3)$

(d) None of the above
(ওপৰৰ এটাও নহয়)

(vi) Euler ϕ function of a prime number p, i.e., $\phi(p) = ?$

এটা মৌলিক সংখ্যা p ৰ আইলাৰ ϕ ফলন অৰ্থাৎ $\phi(p) = ?$

(a) p

(b) $p - 1$

(c) $\frac{p}{2}-1$

(d) None of the above
(ওপৰৰ এটাও নহয়)

(vii) Which theorem states that 'If p is a prime, then $(p-1)! \equiv -1 \,(mod\ p)$' ?

(a) Dirichlet's theorem
(b) Wilson's theorem
(c) Euler's theorem
(d) Fermat's Little theorem

যদি p এটা মৌলিক সংখ্যা হয়, তেতিয়া $(p-1)! \equiv -1 \,(mod\ p)$ — এইটো তলৰ কোনটো উপপাদ্য হ'ব?

(a) Dirichlet ৰ উপপাদ্য
(b) Wilson ৰ উপপাদ্য
(c) Euler ৰ উপপাদ্য
(d) Fermat ৰ Little উপপাদ্য

(viii) "Let p be a prime and p does not divide a, then $a^{p-1} \equiv 1 \,(mod\ p)$" is a statement of

(a) Dirichlet's theorem
(b) Euclid's theorem
(c) Fermat's Little theorem
(d) Wilson's theorem

"ধৰা হ'ল p এটা মৌলিক সংখ্যা আৰু p, a ৰ বিভাজ্য নহয়, তেতিয়াহলে $a^{p-1} \equiv 1 \,(mod\ p)$" এইটো তলৰ কোনটো উপপাদ্যৰ?

(a) Dirichlet ৰ উপপাদ্য
(b) Euclid ৰ উপপাদ্য
(c) Fermat ৰ Little উপপাদ্য
(d) Wilson ৰ উপপাদ্য

(ix) The unit place digit of 3^{46} is
3^{46} ৰ একক স্থানৰ অংকটো হ'ব

(a) 3
(b) 7
(c) 1
(d) 9

(x) The highest power of 7 that divides $50!$ is
$50!$ ৰে বিভাজ্য 7 ৰ সৰ্বোচ্চ ঘাতটো হ'ব

(a) 7
(b) 8
(c) 10
(d) 5

2. Answer the following questions : 2×5=10
তলৰ প্ৰশ্নকেইটাৰ উত্তৰ দিয়া :

(a) Apply Chinese remainder theorem to solve
চাইনীজ ভাগশেষ উপপাদ্য প্ৰয়োগ কৰি সমাধান কৰা

$$x \equiv 3 \,(mod\ 5)$$
$$x \equiv 5 \,(mod\ 7)$$

(b) Find $\phi(100)$.

$\phi(100)$ ৰ মান উলিওৱা।

(c) Prove that $\tau(m)$ is multiplicative, i.e., $\tau(m \cdot n) = \tau(m)\,\tau(n)$.

প্ৰমাণ কৰা যে, $\tau(m)$ পূৰণৰ যোগ্য অৰ্থাৎ $\tau(m \cdot n) = \tau(m)\,\tau(n)$।

(d) Find $\sigma(12)$.

$\sigma(12)$ ৰ মান নিৰ্ণয় কৰা।

(e) Let x and y be any real numbers, prove that $[x]+[y] \le [x+y]$ (where $[x]$ denotes greatest integer $\le x$)

ধৰা হ'ল x আৰু y দুটা যিকোনো বাস্তৱ সংখ্যা। প্ৰমাণ কৰা যে, $[x]+[y] \le [x+y]$ (য'ত $[x]$ এটা সৰ্বোচ্চ অখণ্ড সংখ্যা $\le x$)

3. Answer ***any four*** questions : 5×4=20

তলৰ *যিকোনো চাৰিটা* প্ৰশ্নৰ উত্তৰ কৰা ঃ

(a) Find the remainder when 30^{40} is divided by 17.

30^{40} ক 17 ৰে হৰণ কৰিলে ভাগশেষ নিৰ্ণয় কৰা।

(b) If $d = gcd\,(a, n)$ prove that the linear congruence $ax \equiv b\,(mod\,n)$ has a solution if and only if $d|b$.

যদি $d = gcd\,(a, n)$, প্ৰমাণ কৰা যে $ax \equiv b\,(mod\,n)$ ৰৈখিক congruence ৰ এটা সমাধান থাকিব যদি আৰু একমাত্ৰ যদিহে $d|b$।

(c) If p is prime and $k > 0$, then prove that

যদি p এটা মৌলিক সংখ্যা হয় আৰু $k > 0$ হয়, তেন্তে প্ৰমাণ কৰা যে

$$\phi\left(p^k\right) = p^k - p^{k-1} = p^k\left(1 - \frac{1}{p}\right)$$

Verify this result for $\phi(16)$.

$\phi(16)$ ৰ বাবে ফলটোৰ সত্যাপন কৰা।

(d) For $n = p^k$, p is a prime, prove that $n = \sum_{d|n}\phi(n)$, where $\sum_{d|n}$ denotes the sum over all positive divisors of n.

$n = p^k$, য'ত p এটা মৌলিক সংখ্যা, ৰ বাবে প্ৰমাণ কৰা যে, $n = \sum_{d|n}\phi(n)$ য'ত $\sum_{d|n}$ য়ে n ৰ ধনাত্মক বিভাজ্যসমূহৰ যোগফল নিৰ্দেশ কৰে।

(e) Prove that no integer of the form $4n+3$ is the sum of two squares.

প্ৰমাণ কৰা যে, $4n+3$ আকাৰৰ দুটা বৰ্গৰ যোগফলৰ কোনো অখণ্ড সংখ্যা পাব নোৱাৰি।

(f) Show that an integer $p>1$ is a prime if p divides $a.b$ implies either p divides a or p divides b.

দেখুওৱা যে, এটা অখণ্ড সংখ্যা $p>1$ এটা মৌলিক হ'ব যদিহে $a.b$, p ৰে বিভাজ্যই বুজায় — হয় a, p ৰে বিভাজ্য অথবা b, p ৰে বিভাজ্য।

PART–B

Answer **any four** *from the following questions:* 10×4=40

তলৰ প্ৰশ্নসমূহৰ পৰা যিকোনো চাৰিটা প্ৰশ্নৰ উত্তৰ দিয়া ঃ

4. (a) Show that, the set of integers $\{1, 5, 7, 11\}$ is a RRS (reduced residue system) modulo 12. 5

দেখুওৱা যে, $\{1, 5, 7, 11\}$ অখণ্ড সংখ্যাৰ সংহতিটো এটা modulo 12 ৰ RRS।

(b) If a, b, c be integers such that $ac \equiv bc \,(mod\, m)$ and $d \equiv gcd\,(c, m)$, then prove that $a \equiv b \left(mod\, \frac{m}{d}\right)$. 5

যদি a, b, c অখণ্ড সংখ্যা হয়, যাতে $ac \equiv bc\,(mod\, m)$ আৰু $d \equiv gcd\,(c, m)$ তেতিয়াহলে, প্ৰমাণ কৰা যে $a \equiv b\left(mod\, \frac{m}{d}\right)$.

5. (a) If p is a prime, then prove that

যদি p এটা মৌলিক, তেন্তে প্ৰমাণ কৰা যে

$$\phi(p!) = (p-1)\,\phi((p-1)!)$$ 5

(b) Prove that $5n+3$ and $7n+4$ are co-prime to each other for any natural number n. 5

প্ৰমাণ কৰা যে যিকোনো স্বাভাৱিক সংখ্যা n ৰ বাবে $5n+3$ আৰু $7n+4$ সংখ্যা দুটা এটা আনটোৰ সহ-মৌলিক হ'ব।

6. (a) If p is a prime, then prove that $(p-1)! \equiv -1\,(mod\, p)$. 5

যদি p এটা মৌলিক হয়, তেন্তে প্ৰমাণ কৰা যে $(p-1)! \equiv -1\,(mod\, p)$.

(b) Solve the following simultaneous congruence : 5

$$x \equiv 3\,(mod\, 11)$$
$$x \equiv 5\,(mod\, 19)$$
$$x \equiv 10\,(mod\, 29)$$

তলৰ সহ congruence কেইটা সমাধান কৰা ঃ

$$x \equiv 3 \pmod{11}$$
$$x \equiv 5 \pmod{19}$$
$$x \equiv 10 \pmod{29}$$

7. (a) Using property of congruence, show that, 41 divides $2^{20}-1$. 5

Congruence ৰ ধৰ্ম ব্যৱহাৰ কৰি দেখুওৱা যে $2^{20}-1$, 41 ৰে বিভাজ্য।

(b) Prove that any positive integer $n\,(n>1)$ can be expressed uniquely as a product of primes. 5

প্ৰমাণ কৰা যে যিকোনো ধনাত্মক অখণ্ড সংখ্যা $n\,(n>1)$ ক মৌলিক সংখ্যাৰ পূৰণফল হিচাপে একক ৰূপত প্ৰকাশ কৰিব পাৰি।

8. (a) If n is any integer can be expressed as $n = p_1^{\alpha_1}.p_2^{\alpha_2}.....p_k^{\alpha_k}$, then prove that

$$\phi(n) = n\prod_{j=1}^{k}\left(1-\frac{1}{p_j}\right)$$ 5

যদি যিকোনো অখণ্ড সংখ্যা n ক, $n = p_1^{\alpha_1}.p_2^{\alpha_2}.....p_k^{\alpha_k}$ ৰে প্ৰকাশ কৰিব পাৰি, তেন্তে প্ৰমাণ কৰা যে, $\phi(n) = n\prod_{j=1}^{k}\left(1-\frac{1}{p_j}\right)$

(b) If m and n are any two integers such that $(m,n)=1$, prove that

$$\phi(m,n) = \phi(m).\phi(n)$$ 5

যদি m আৰু n দুটা অখণ্ড সংখ্যা হয় যাতে $(m,n)=1$, তেন্তে প্ৰমাণ কৰা যে

$$\phi(m,n) = \phi(m).\phi(n)$$

9. (a) If $n = p_1^{k_1}.p_2^{k_2}.....p_r^{k_r}$ is the prime factorization of $n>1$, then prove that

যদি $n = p_1^{k_1}.p_2^{k_2}.....p_r^{k_r}$ এটা $n>1$ ৰ মৌলিক উৎপাদক হয়, তেন্তে প্ৰমাণ কৰা যে

$$\tau(n) = (k_1+1)(k_2+1)\;.....\;(k_r+1)$$

$$\sigma(n) = \frac{p_1^{k_1+1}-1}{p_1-1}\times\frac{p_2^{k_2+1}-1}{p_2-1}\times.....\times\frac{p_r^{k_r+1}-1}{p_r-1}$$ 5

(b) Find the number of zeroes in the end of the product of first 100 natural numbers. 5

প্ৰথম 100 টা স্বাভাৱিক সংখ্যাৰ পূৰণফলৰ শেহত থকা শূন্যৰ সংখ্যা নিৰ্ণয় কৰা।

10. (a) Define Mobiu's function. Also prove that

$$\mu(m \cdot n) = \mu(m) \cdot \mu(n)$$

Hence find $\mu(6)$. 5

মবিয়াৰ ফলনৰ সংজ্ঞা দিয়া। প্ৰমাণ কৰা যে, $\mu(m \cdot n) = \mu(m) \cdot \mu(n)$

ইয়াৰ পৰা $\mu(6)$ ৰ মান উলিওৱা।

(b) For each positive integer $n \geq 1$, show that

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases}$$ 5

প্ৰতিটো ধনাত্মক অখণ্ড সংখ্যা $n \geq 1$ ৰ বাবে দেখুওৱা যে $\sum_{d|n} \mu(d) = \begin{cases} 1, & \text{যদি } n = 1 \\ 0, & \text{যদি } n > 1 \end{cases}$

11. (a) Let $a,\ m > 0$ be integers such that $(a, m) = 1$, then prove that

$$a^{\phi(m)} \equiv 1 \,(mod\ m)$$ 5

যদিহে $a,\ m > 0$ অখণ্ড সংখ্যা হয় যʼত $(a, m) = 1$, তেন্তে প্ৰমাণ কৰা যে

$$a^{\phi(m)} \equiv 1 \,(mod\ m)$$

(b) Solve (সমাধান কৰা) : 5

$$f(x) = x^2 + x + 7 \equiv 0 \,(mod\ 3)$$

OPTION–B

Paper : MAT–RE–5026

(Discrete Mathematics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Give very short answers : $1 \times 10 = 10$

(a) Define upper and lower bound of a poset.

(b) What is a chain ?

(c) Write the commutative law of a lattice.

(d) Define a sublattice.

(e) Every singleton set of a lattice is a sublattice. State true **or** false.

(f) State De Morgan's law.

(g) Write the identity law of a Boolean algebra.

(h) Define a Boolean variable.

(i) Define a complemented lattice.

(j) Write the basic operations of a Boolean algebra.

2. Give short answers : 2×5=10

(a) Let $A=\{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered set with the relation 'x divides y'. Draw its Hasse diagram.

(b) Write the properties of a distributive lattice.

(c) Give an example of a modular lattice.

(d) Prove that in a Boolean algebra B, $(a')' = a, \ \forall a \in B$.

(e) If L be a lattice, then for every $a, b \in L$, prove that $a \vee b = b$ iff $a \leq b$.

3. Answer the following : ***(any four)*** 5×4=20

(a) Show that $(Z^+, /)$ is a lattice.

(b) Express the Boolean function $f(x, y, z) = x + y'z$ in a sum of minterms.

(c) In a distributive lattice $(L, <)$ prove that $a \wedge b = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$.

(d) Give an example to show that the union of two sublattices may not be a sublattice.

(e) Express the Boolean function $f(x, y, z) = xy + x'y$ as product of maxterms.

(f) Prove that the elements 0 and 1 of Boolean algebra are unique.

4. Answer the following : ***(any four)*** 10×4=40

(a) Prove that a non-empty finite poset has

(i) at most one greatest element;

(ii) at most one least element.

(b) Show that in a complemented distributive lattice the following are equivalent :

(i) $a \leq b$

(ii) $a \wedge b' = 0$

(iii) $a' \vee b = 1$

(iv) $b' \leq a'$

(c) Consider the subsets $\{2, 3\}$, $\{4, 6\}$, $\{3, 6\}$ of the poset $\{\{1, 2, 3, 4, 5, 6\}, /\}$. Find for each subset (if exist)

(i) upper bound and lower bound;

(ii) greatest lower bound and least upper bound.

(d) Write a short note on Karnaugh map method in Boolean algebra. Find the Karnaugh maps and simplify the following expressions :

(i) $AB' + A'B'$

Contd.

(ii) $AB' + A'B$

(iii) $AB' + A'B + A'B'$

(e) Simplify the following Boolean expressions :

(i) $x(y+z)(x+y+z)$

(ii) $xy + x'z + yz$

(iii) $x+y(x+y)+x(x'+y)$

(f) Determine all the sublattices of D_{30} containing four elements.

$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Also draw the Hasse diagram.

(g) In a Boolean algebra B, prove the following :

(i) $(a+b)' = a'.b', \ \forall a, b \in B$

(ii) $(a.b)+(a'+b')=1, \ \forall a, b \in B$

(iii) $(a+b)(a'+c)=ac+aB, \ \forall a, b, c \in B$

(h) Express the following in disjunctive normal form : *(any two)*

(i) $f(x, y, z)=(yz+xz')(xy'+z)'$

(ii) $f(x, y, z)=(x+x'y+x'yz')\left(xy+(xz)'\right)(y+xyz')$

(iii) $f(x, y, z)=(x'y)'.(x+y)$

Total number of printed pages–24

3 (Sem–5/CBCS) MAT HE 1/HE 2/HE 3

2021

(Held in 2022)

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5016

(Number Theory)

DSE (H)–1

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

PART–A

1. Choose the correct option : $1 \times 10 = 10$

   (i) Two integers a and b are coprime if there exists some integers x, y such that

   (a) $ax + by = 1$

Contd.

(b) $ax - by = 1$

(c) $(ax + by)^n = 1$

(d) None of the above

(ii) Let $d = gcd(a, b), n \in \mathbb{N}$. If $d \mid c$ and (x_0, y_0) is a solution of linear Diophantine equation $ax + by = c$, then all integral solutions are given by

(a) $(x, y) = \left(x_0 + \frac{bn}{d}, y_0 - \frac{an}{d}\right)$

(b) $(x, y) = \left(x_0 - \frac{bn}{d}, y_0 + \frac{an}{d}\right)$

(c) $(x, y) = \left(x_0 + \frac{an}{d}, y_0 - \frac{bn}{d}\right)$

(d) $(x, y) = \left(x_0 - \frac{an}{d}, y_0 + \frac{bn}{d}\right)$

(iii) A reduced residue system modulo m is a set of integers r_i such that

(a) $[r_i, m] = 1$

(b) $(r_i, m) = 1$

(c) $(r_i, m) \neq 1$

(d) None of the above

(iv) Suppose that m_j are pairwise relatively prime and a_j are arbitrary integers $(j = 1, 2, \dots k)$ then there exist solution x to the simultaneous congruence $x \equiv a_j (mod\ m_j)$, such that x are

(a) congruent modulo $M = m_1.m_2.m_3 \dots m_k$

(b) congruent modulo $M = \sum_{j=1}^{k} m_j$

(c) congruent modulo m_i

(d) Both (a) and (b)

(v) The product of four consecutive positive integers is divisible by

(a) 20

(b) 22

(c) 24

(d) 26

(vi) Euler's ϕ-function of a prime number p, i.e., $\phi(p)$ is

(a) p

(b) $p-1$

(c) $\frac{p}{2} - 1$

(d) None of the above

Contd.

(vii) For which value of m,
CRS $(mod\ m)$ = RRS $(mod\ m)$?

(a) If m is a prime

(b) If m is a composite

(c) If $m < 10$

(d) None of the above

(viii) If $ca \equiv cb(mod m)$, then

(a) $a \equiv b\left(mod \frac{m}{(c, m)}\right)$

(b) $a \equiv b(mod m)$

(c) $a \equiv b(mod m. (c, m))$

(d) None of the above

(ix) The unit place digit of 2^{73} is

(a) 4

(b) 6

(c) 8

(d) 2

(x) The highest power of 7 that divides 50 ! is

(a) 7

(b) 8

(c) 10

(d) 5

2. Answer the following questions : 2×5=10

(a) If p is a prime, then prove that $\phi(p!) = (p-1)\,\phi((p-1)!)$ 2

(b) Find all prime number p such that $p^2 + 2$ is also a prime. 2

(c) For $n = p^k$, p is a prime, prove that
$$n = \sum_{d|n} \phi(d)$$
where $\sum_{d|n}$ denotes the sum over all positive divisors of n. 2

(d) Find the number of zeros at the end of the product of first 100 natural numbers. 2

(e) Find $\sigma(12)$. 2

3. Answer ***any four*** questions : 5×4=20

(a) If ϕ is Euler's phi function, then find $\phi(\phi(1001))$. 5

(b) Find the remainder, when 30^{40} is divided by 17. 5

(c) State and prove Chinese Remainder Theorem. 5

(d) If p_n is the nth prime number, then prove that

$$p_n < 2^{2^{n-1}}$$ 5

(e) If $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_r^{k_r}$ is the prime factorization of $n > 1$, then prove that

(i) $\tau(n) = (k_1+1)(k_2+1)(k_3+1)\dots(k_n+1)$

(ii) $\sigma(n) = \dfrac{p_1^{k_1+1}-1}{p_1-1} \times \dfrac{p_2^{k_2+1}-1}{p_2-1} \times \dots \times \dfrac{p_r^{k_r+1}-1}{p_r-1}$

2½+2½=5

(f) Define Mobius function. Also show that

$$\mu(m \cdot n) = \mu(m) \cdot \mu(n)$$

Hence find $\mu(6)$. 1+3+1=5

PART–B

*Answer **any four** questions :* 10×4=40

4\. (a) If $d = (a, n)$, prove that the linear congruence $ax \equiv b \pmod{n}$ has a solution if and only if $d \mid b$. 5

(b) (i) When a number n is divided by 3 it leaves remainder 2. Find the remainder when $3n+6$ is divided by 3. 2

(ii) Prove that $5n+3$ and $7n+4$ are coprime to each other for any natural number n. 3

5\. (a) If p is a prime, then prove that

$$(p-1)! \equiv -1 \pmod{p}$$ 5

(b) Using property of congruence show that 41 divides $2^{20}-1$. 5

6\. (a) Prove that every positive integer $(n > 1)$ can be expressed uniquely as a product of primes. 5

(b) Determine all solutions in the integers of the Diophantine equation

$$172x + 20y = 1000$$ 5

7\. (a) If n be any positive integer and can be expressed as $n = p_1^{\alpha_1} . p_2^{\alpha_2} \dots\dots p_k^{\alpha_k}$, then prove that $\phi(n) = n \prod_{j=1}^{k} \left(1 - \dfrac{1}{p_j}\right)$. 5

(b) If m and n are any two integers such that $(m, n) = 1$, prove that

$$\phi(m.n) = \phi(m).\phi(n).$$ 5

8. (a) For each positive integer $n \geq 1$, show that

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases}$$ 5

(b) If k denotes the number of distinct prime factors of positive integer n, then prove that

$$\sum_{d|n} |\mu(d)| = 2^k$$ 5

9. (a) Show that $\sum_{d|n} \mu(d)\,\tau(d) = (-1)^k$

where k denotes the number of distinct prime factors of positive integers n. 5

(b) Prove that

(i) $\tau(n)$ is an odd integer iff n is a perfect square. 3

(ii) For any integer $n \geq 3$, show that

$$\sum_{k=1}^{n} \mu(k!) = 1.$$ 2

10. (a) Let p be an odd prime. Show that the congruence $x^2 \equiv -1 (mod\, p)$ has a solution if and only if $p \equiv 1 (mod\ 4)$. 5

(b) If $n \geq 1$ and $gcd(a, n) = 1$, then prove that $a^{\phi(n)} \equiv 1 (mod\, n)$. 5

11. (a) If n is a positive integer and p is a prime, then prove that the exponent of the highest power of p that divides $n!$

is $\sum_{k=1}^{\infty} \left[\frac{n}{p^k}\right]$. 5

(b) Solve $3[x] = x + 2\{x\}$ where $[x]$ denotes greatest integer $\leq x$ and $\{x\}$ denotes the fractional part of x. 5

OPTION–B

Paper : MAT–HE–5026

(Mechanics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (i) What is the physical significance of the moment of a force?

   (ii) State Newton's second law of motion.

   (iii) Define angle of friction.

   (iv) Define the centre of gravity of a body.

   (v) What do you mean by terminal velocity?

   (vi) What is the geometrical representation of the simple harmonic motion?

   (vii) What is the length of arm of a couple equivalent to the couple (P, p) having constituent force of magnitude F?

   (viii) Can a force and a couple in the same plane be equivalent to a single force?

   (ix) Define a couple.

   (x) State Hooke's law.

2. Answer the following questions : 2×5=10

   (a) Find the greatest and least resultant of two forces acting at a point whose magnitudes are P and Q respectively.

   (b) Find the centre of gravity of an arc of a plane curve $y = f(x)$.

   (c) State the laws of static friction.

   (d) Show that impulse of a force is equal to the momentum generated by the force in the given time.

   (e) Write the expression for the component of velocity and acceleration along radial and cross-radial direction for a motion of a particle in a plane curve.

3. Answer ***any four*** questions of the following : 5×4=20

(a) The line of action of a force F divides the angle between its component forces P and Q in the ratio $1:2$. Prove that $Q(F+Q)=P^2$.

(b) P and Q are two like parallel forces. If P is moved parallel to itself through a distance x, show that the resultant of P and Q moves through a distance $\frac{Px}{P+Q}$.

(c) R is the resultant of two forces P and Q acting at a point and at a given angle. If the force P be doubled, show that the new resultant will be of magnitude $\sqrt{2(P^2+R^2)-Q^2}$.

(d) A particle of mass m moves in a straight line under acceleration mn^2x towards a point O on the line, where x is the distance from O. Show that if $x=a$ and $\frac{dx}{dt}=u$ when $t=0$, then at time t,

$$x=a\cos nt+\frac{u}{n}\sin nt.$$

(e) A particle moving with simple harmonic motion in a straight line has velocity v_1 and v_2 at distance x_1, x_2 from the centre of its path. Show that if T be the period of its motion then $T=2\pi\sqrt{\frac{x_1^2-x_2^2}{v_2^2-v_1^2}}$.

(f) Show that the sum of the kinetic energy and potential energy is constant throughout the motion when a particle of mass m falls from rest at a height h above ground.

4. Answer ***any four*** questions of the following : 10×4=40

(a) Forces P, Q and R act along the sides BC, CA and AB of a triangle ABC and forces P', Q' and R' act along OA, OB and OC, where O is the centre of the circumscribed circle, prove that

(i) $P\cos A+Q\cos B+R\cos C=0$

(ii) $\frac{PP'}{a}+\frac{QQ'}{b}+\frac{RR'}{c}=0$

(b) State and prove Lami's theorem. Forces P, Q and R acting along OA, OB and OC, where O is the circumcentre of triangle ABC, are in equilibrium. Show that

$$\frac{P}{a^2\left(b^2+c^2-a^2\right)}=\frac{Q}{b^2\left(c^2+a^2-b^2\right)}=\frac{R}{c^2\left(a^2+b^2-c^2\right)}$$

(c) (i) Find the centre of gravity of a circular arc of radius a which subtends an angle 2α at the centre.

(ii) Find the centre of gravity of a uniform parabolic area cut off by a double ordinate at a distance h from the vertex.

(d) (i) Show that the least force which will move a weight W along a rough horizontal plane is $W \sin\phi$, where ϕ is the angle of friction.

(ii) If a body is placed upon a rough inclined plane, and is on the point of sliding down the plane under the action of its weight and the reactions of the plane only, show that the angle of inclination of the plane to the horizon is equal to the angle of friction.

(e) A particle moves in a straight line under an attraction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Investigate the motion.

(f) A particle moves in a straight line OA starting from the rest at A and moving with an acceleration which is directed towards O and varies as the distance from O. Discuss the motion of the particle. Hence define simple harmonic motion and time period of the motion.

(g) Find the component of acceleration of a point moving in a plane curve along the initial line and the radius vector. Also find the component of acceleration perpendicular to initial line and perpendicular to radius vector.

(h) A particle is falling under gravity in a medium whose resistance varies as the velocity. Find the distance and velocity at any time t. Also find the terminal velocity of the particle.

OPTION–C

Paper : MAT–HE–5036

(Probability and Statistics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

(a) Write the sample space for the experiment of tossing a coin three times in succession or tossing three coins at a time.

(b) Is the probability mass function

x	-1	0	1
$P(x)$	0.3	0.4	0.4

admissible? Give reason.

(c) Sketch the area under any probability curve with probability function $p(x)$ between $x = c$ and $x = d$ represented by

$$P(c \le X \le d) = \int_c^d p(x)\, dx.$$

(d) What conclusion one can make about the conditional probability $P(A/B)$ if $P(B) = 0$?

(e) State the multiplicative theorem of expectation.

(f) Mention the relationship among the mean, median and mode of the normal distribution.

(g) If X and Y are two independent random variables, then find $Var(2X + 3Y)$.

(h) Write the mean and variance of standard normal variate $Z = \frac{X - \mu}{\sigma}$, where μ and σ are mean and standard deviation respectively.

(i) When is the correlation coefficient between two random variables X and Y zero.

(j) State weak law of large number.

2. Answer the following questions : 2×5=10

(a) Prove that probability of any impossible event is zero.

(b) If X is a random variable, then prove that $Var(X) = E(X^2) - \{E(X)\}^2$

(c) Find the constant c such that the function

$$f(x) = cx^2, \; 0 < x < 3$$
$$\quad\;\; 0, \text{ otherwise.}$$

is a density function and also find $P(0 < x < 3)$.

(d) A random variable X has density function given by

$$f(x) = 2e^{-2x}, \; x \geq 0$$
$$\quad\;\; 0, \quad \text{otherwise}$$

then find the moment generating function.

(e) Comment on the following statement : "The mean of a binomial distribution is 3 and its standard deviation is 2".

3. Answer **any four** parts from the following : 5×4=20

(a) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second to give 4 black balls if the balls are not replaced before the second draw.

(b) For two independent events A and B prove that (i) A and B are independent, and (ii) $\overline{A}$ and $\overline{B}$ are independent.

(c) A random variable X has the function

$f(x) = \dfrac{c}{x^2 + 1}$, where $-\infty < x < \infty$, then

(i) find the value of the constant c;

(ii) find the probability that X^2 lies between $\frac{1}{3}$ and 1.

(d) The joint probability of two variables X and Y is given by

$$f(x, y) = \frac{1}{42}(2x + y), \; 0 \leq x \leq 2, \; 0 \leq y \leq 3$$
$$\quad\; 0 \quad\quad , \text{ otherwise}$$

Find (i) $F(Y/2)$, and (ii) $P(y = 1/x = 3)$

(e) A coin is tossed until a head appears. What is the expectation of the number of tosses required?

(f) The probability of a man hitting a target is $\frac{1}{4}$.

(i) If he fires 7 times, what is the probability of his hitting the target at least twice?

(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$?

4. Answer **any four** parts from the following: 10×4=40

(a) For n events $A_1, A_2, A_3, \ldots\ldots, A_n$, prove that

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i-1}^{n} P(A_i) - \sum_{1\le i<j\le n}\sum P(A_i \cap A_j) + \sum_{1\le i<j<k\le n}\sum\sum P(A_i \cap A_j \cap A_k) + \ldots\ldots\ldots\ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap A_3 \ldots\ldots \cap A_n)$$

Hence find $P\left(\bigcup_{i=1}^{3} A_i\right)$

(b) Suppose that two dimensional continuous random variables (X, Y) has joint p.d.f given by

$$f(x, y) = 6x^2y,\ 0 < x < 1,\ 0 < y < 1$$
$$0\ ,\ \text{elsewhere}$$

(i) Verify that $\int_0^1\int_0^1 f(x, y)\, dx\, dy = 1$

(ii) Find $P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right)$

(iii) Find $P(X + Y < 1)$

(iv) Find $P(X > Y)$

(v) Find $P(X < 1/Y < 2)$

(c) (i) The probability function of a random variable X is given by

$$f(x, y) = \frac{x^2}{81}, -3 < x < 6$$
$$0\ ,\ \text{otherwise}$$

Find the probability density function for the random variable $u = \frac{1}{3}(12 - X)$.

Contd.

(ii) Define moment generating function of a random variable X. Find the moment generating function of binomial distribution.

(d) (i) The probability curve $y = f(x)$ has a range from 0 to ∞. If $f(x) = e^{-x}$, find the mean and variance.

(ii) If X be a continuous random variable with probability density function

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

compute $P(X \le 1.5)$

(e) (i) Derive Poisson distribution as a limiting case of binomial distribution.

(ii) Prove that mean and variance of a binomially distributed variable are respectively np and npq.

(f) (i) Define correlation coefficient of two random variables X and Y. Show that correlation coefficient is independent of change of origin and scale.

(ii) Obtain the equation of two lines of regression for the following data :

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

Also obtain the estimate of X for $Y = 70$.

(g) (i) If X is a random variable with mean μ and variance σ^2, then for any positive number k, prove that

$$P\{|X - \mu| \ge k\sigma\} \le \frac{1}{k^2}$$

(ii) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.

(h) The random variables X and Y have the following joint probability density function :

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find—

(i) marginal probability density functions of X and Y;

Contd.

(ii) conditional density functions ;

(iii) $Var(X)$;

(iv) $Var(Y)$;

(v) covariance between X and Y.

Total number of printed pages–7

3 (Sem–5/CBCS) MAT SE

2021

(Held in 2022)

MATHEMATICS

(Skill Enhancement Course)

Paper : MAT–SE–5014

(Combinatorics and Graph Theory)

Full Marks : 50

Time : Two hours

The figures in the margin indicate full marks for the questions.

Answer ***either*** *in English* ***or*** *in Assamese.*

1. Answer ***any four*** of the following : $1 \times 4 = 4$

তলৰ *যিকোনো চাৰিটা* প্ৰশ্নৰ উত্তৰ কৰা :

(a) State the pigeonhole principle.

পিজনহল সূত্ৰটো লিখা।

(b) State the disjunctive or sum rule in counting.

গণনাৰ যোগৰ সূত্ৰটো লিখা।

(c) A college campus has four gates. You can enter to the campus through a gate and can exit through a different gate. In how many ways you can complete the entry-exit process?

কলেজ এখনৰ চৌহদত চাৰিটা গেট আছে। তুমি এবাৰ এখন গেটেৰে প্ৰবেশ কৰিলে সেইবাৰ বেলেগ এখন গেটেৰে প্ৰস্থান কৰিব লাগিব। তুমি কিমান ভিন্ন ধৰণেৰে প্ৰবেশ-প্ৰস্থান কৰিব পাৰিবা ?

(d) Write true **or** false :

"Union of two distinct paths joining two points contains a cycle."

সঁচা *নে* মিছা লিখা :

"দুটা বিন্দু সংযোগী দুটা ভিন্ন পথৰ মিলনত এটা চক্ৰ থাকে।"

(e) Draw all possible graphs with *three* points.

তিনিটা বিন্দু থকা সম্ভাব্য সকলোবোৰ লেখ অংকন কৰা।

2. Answer ***any three*** of the following : 2×3=6

তলৰ *যিকোনো তিনিটা* প্ৰশ্নৰ উত্তৰ কৰা :

(a) Give the difference between a connected graph and a complete graph with diagrams.

এটা সংযুক্ত লেখ আৰু এটা সম্পূৰ্ণ লেখৰ মাজৰ পাৰ্থক্য ছবিৰ সহায়ত উল্লেখ কৰা।

(b) G is a graph with p points and q lines. H is a graph with r points and s lines. Then find the number of points and lines in $G+H$ and $G \times H$.

p টা বিন্দু আৰু q টা বাহু যুক্ত G এটা লেখ। rটা বিন্দু আৰু s টা বাহু যুক্ত H এটা লেখ। $G+H$ আৰু $G \times H$ত বিন্দু আৰু বাহুৰ সংখ্যা নিৰ্ণয় কৰা।

(c) Determine the number of diagonals formed by joining the angular points of a decagon.

এটা দশভূজৰ শীৰ্ষবিন্দুকেইটা সংযোগ কৰি কিমানডাল কৰ্ণ পাব পাৰি ?

(d) In how many ways n girls and n boys can be seated in a row of $2n$ chairs if two boys or two girls cannot seat together?

n জন লৰা আৰু n জনী ছোৱালীক এটা শাৰীত থকা $2n$ খন চকীত কিমান ধৰণে বহুৱাব পাৰি যদি দুজন লৰা বা দুজনী ছোৱালীৰে একেলগে নবহে ?

3. Answer ***any two*** of the following : 5×2=10

তলৰ *যিকোনো দুটা* প্ৰশ্নৰ উত্তৰ কৰা :

(a) Show that a complete graph with n vertices consists of $n(n-1)/2$ edges.

দেখুওৱা যে n শীৰ্ষবিন্দুযুক্ত সম্পূৰ্ণ লেখ এটাত $n(n-1)/2$ টা বাহু থাকে।

(b) Show that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

where n, r are positive integers.

দেখুওৱা যে $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

যত n, r ধনাত্মক অখণ্ড সংখ্যা।

(c) Let G be a simple graph with at most $2n$ vertices. If the degree of each vertex is at least n, then show that the graph G is connected.

$2n$ সৰ্বোচ্চ সংখ্যক শীৰ্ষবিন্দুযুক্ত G এটা সৰল লেখ। যদি প্ৰতিটো শীৰ্ষবিন্দুৰ ঘাত অন্ততঃ n হয়, তেন্তে দেখুওৱা যে G লেখটো এটা সংযোগী লেখ।

(d) How many positive integer solutions are there in the equation

$x+y+z=13$ such that $x, y \leq 4$, $z \leq 6$?

$x+y+z=13$ যত $x, y \leq 4$, $z \leq 6$ সমীকৰণটোৰ ধনাত্মক অখণ্ড সমাধানৰ মুঠ সংখ্যা কিমান ?

4. Answer ***any three*** of the following : 10×3=30

তলৰ *যিকোনো তিনিটা* প্ৰশ্নৰ উত্তৰ কৰা :

(a) Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

দেখুওৱা যে ত্ৰিভুজ নথকা p বিন্দুযুক্ত লেখ এটাত থকা সৰ্বোচ্চ বাহুৰ সংখ্যা $\left[\frac{p^2}{4}\right]$.

(b) Show that any set of seven distinct integers contains two integers x and y such that either $x+y$ or $x-y$ is divisible by 10.

দেখুওৱা যে সাতটা ভিন্ন অখণ্ড সংখ্যাৰ সংহতি এটাত দুটা অখণ্ড সংখ্যা x আৰু y থাকে যাতে $x+y$ বা $x-y$ ৰ মান 10 ৰে বিভাজ্য হয়।

(c) Show that if G is a linear graph of n vertices such that the sum of the degrees for each pair of vertices in G is $(n-1)$ or larger, then there exists a Hamiltonian path in G.

দেখুওৱা যে যদি G এটা n টা শীৰ্ষবিন্দুযুক্ত বৈখিক লেখ হয় যাৰ প্ৰতিযোৰ শীৰ্ষবিন্দুৰ ঘাতৰ সমষ্টি $(n-1)$ বা তাতকৈ অধিক হয়, তেন্তে G ত এটা হেমিল্টনীয়ান পথ থাকিব।

(d) Find the number of integers between 1 and 250 that are divisible by *any* of the integers 2, 3 and 7.

2, 3 আৰু 7ৰ *যিকোনো* এটাৰে বিভাজ্য, 1 আৰু 250ৰ মাজত থকা অখণ্ড সংখ্যাৰ মুঠ সংখ্যা নিৰ্ণয় কৰা।

(e) *(i)* Show that a simple graph with *at least two* vertices has *at least two* vertices of same degree.

দেখুওৱা যে অন্ততঃ *দুটা* শীৰ্ষবিন্দুযুক্ত এটা সৰল লেখৰ সমান ঘাতৰ অন্ততঃ *দুটা* শীৰ্ষবিন্দু থাকিব।

(ii) Show that in a graph, there are even number of vertices of odd degree.

দেখুওৱা যে এটা সৰল লেখৰ অযুগ্ম ঘাতৰ শীৰ্ষবিন্দুৰ সংখ্যা যুগ্ম হ'ব।

(f) Give definition with an example of each of the following :

এটাকৈ উদাহৰণেৰে সৈতে তলত দিয়াবোৰৰ সংজ্ঞা দিয়া :

(i) Pseudograph

কূটলেখ

(ii) Product rule of counting

গণনাৰ পূৰণৰ সূত্ৰ

(iii) The problem of Ramsey

ৰমছীৰ সমস্যা

(iv) Complete graph

সম্পূৰ্ণ লেখ

(v) Complement of a graph

লেখ এটাৰ পূৰক

Total number of printed pages–16

3 (Sem–5/CBCS) MAT HE 4/5/6

2021

(Held in 2022)

MATHEMATICS

(Honours Elective)

Answer the Questions from any one Option.

OPTION–A

Paper : MAT–HE–5046

(Linear Programming)

DSE(H)–2

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1\times10=10$

   (a) A basic feasible solution whose variables are.

   (i) degenerate

   (ii) nondegenerate

Contd.

(iii) non-negative

(iv) None of the above

(Choose the correct answer)

(b) The inequality constraints of an LPP can be converted into equation by introducing

(i) negative variables

(ii) non-degenerate B.F.

(iii) slack and surplus variables

(iv) None of the above

(Choose the correct answer)

(c) A solution of an LPP, which optimize the objective function is called

(i) basic solution

(ii) basic feasible solution

(iii) optimal solution

(iv) None of the above

(Choose the correct answer)

(d) What is artificial variable of an LPP ?

(e) Write the equation of line segment in $\mathbb{R}^n$.

(f) Define dual of a given LPP.

(g) What is pure strategy of game theory ?

(h) Is region of feasible solution to an LPP constitute a convex set ?

(i) Is every convex set in $\mathbb{R}^n$ a convex polyhedron also ?

(j) Is every boundary point an extreme point of a convex set ?

2. Answer the following questions : $2\times5=10$

(a) Show that the feasible solution $x_1=1, x_2=0, x_3=1, z=6$ to the system

$$\min\ Z=2x_1+3x_2+4x_3$$

$$\text{s.t.}\quad x_1+x_2+x_3=2$$

$$x_1-x_2+x_3=2\ ,\quad x_i\geq 0$$

is not basic.

(b) A hyperplane is given by the equation

$$3x_1+2x_2+4x_3+7x_4=8$$

Find in which half space do the point $(-6,1,7,2)$ lie.

(c) Find extreme points if any of the set $S=\{(x,y):\ |x|\leq 1,\ |y|\leq 1\}$

(d) Show by an example that the union of two convex sets is not necessarily a convex set.

(e) If $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ a BFS of the LPP

$$\max\ Z = x_1 + 2x_2 + 4x_3$$

$$\text{s.t.}\quad 2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$
$$x_1, x_2, x_3 \geq 0$$

? Explain.

3. Answer ***any four*** questions : 5×4=20

(a) Prove that the set of all feasible solutions of an LPP is a convex set.

(b) Sketch the convex polygon spanned by the following points in a two-dimensional Euclidean space. Which of these points are vertices ? Express the other as the convex linear combination of the vertices

$$(0,0),\ (0,1),\ (1,0),\ \left(\frac{1}{2},\frac{1}{4}\right).$$

(c) If $x_0 \in S$ where S is the set of all FS of the LPP min $Z = cx$, such that $Ax = b$, $x \geq 0$ minimize the objective function $Z = cx$, then show that x_0 also maximize the objective function $Z^* = (-c)x$ over S.

(d) Find the dual of the following LPP :

$$\min\ Z_p = x_1 + x_2 + x_3$$

$$\text{s.t.}\quad x_1 - 3x_2 + 4x_3 = 5$$
$$2x_1 - 3x_2 \leq 3$$
$$2x_2 - x_3 \geq 5$$
$$x_1, x_2, x_3 \geq 0$$

(e) Prove that the dual of a dual is a primal problem itself.

(f) Write the characteristics of an LPP in canonical form.

4. Answer *(a)* **or** *(b)*, *(c)* **or** *(d)*, *(e)* **or** *(f)*, *(g)* **or** *(h)* : 10×4=40

(a) Old hens can be bought for Rs. 2 each but young ones cost Rs. 5 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week, each being worth 30 paise. A hen costs Re. 1 per week to feed. If I have only Rs. 80 to spend for hens, how many of each kind shall I buy to give a profit of more than Rs. 6 per week, assuming that I can not house more than 20 hens ? Formulate the LPP and solve by graphical method.

(b) Find all basic and then all the basic feasible solutions for the equations

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

and determine the associated general convex combination of the extreme point solutions.

(c) State and prove the fundamental theorem of LPP.

(d) Solve by simplex method :

$$\begin{aligned} \max \quad & Z = 3x_1 + 5x_2 + 4x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 8 \\ & 3x_1 + 2x_2 + 4x_3 \le 15 \\ & 2x_2 + 5x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

(e) If in an assignment problem, a constant is added or substracted to every element of a row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimizes the total cost for the other matrix.

(f) Solve the following transportation problem :

To

From		S_1	S_2	S_3	S_4	Supply
	O_1	1	2	1	4	30
	O_2	3	3	2	1	50
	O_3	4	2	5	9	20
Demand		20	40	30	10	100

(g) For any zero-sum two-persons game where the optimal strategies are not pure and for which A's pay-off matrix is

$$\begin{array}{ccc|cc|} & & & B & \\ & & & \text{I}y_1 & \text{II}y_2 \\ \cline{4-5} A & x_1\text{I} & & a_{11} & a_{12} \\ & x_2\text{II} & & a_{21} & a_{22} \\ \cline{4-5} \end{array}$$

the optimal strategies are (x_1, x_2) and (y_1, y_2) then prove that

$$\frac{x_1}{x_2} = \frac{a_{22} - a_{21}}{a_{11} - a_{12}} \quad \text{and} \quad \frac{y_1}{y_2} = \frac{a_{22} - a_{12}}{a_{11} - a_{21}} \quad \text{and}$$

the value of the game to A is given by

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

(h) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

OPTION–B

Paper : MAT–HE–5056

(*Spherical Trigonometry and Astronomy*)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) State *one* fundamental difference between a spherical triangle and a plane triangle.

   (b) Define primary circle.

   (c) Define polar triangle and its primitive triangle.

   (d) State the third law of Kepler.

   (e) Explain what is meant by rising and setting of stars.

   (f) Write *any two* coordinate systems to locate the position of a heavenly body on the celestial sphere.

   (g) Define synodic period of a planet.

   (h) Mention *one* property of pole of a great circle.

   (i) Just mention how a spherical triangle is formed.

   (j) What is the declination of the pole of the ecliptic ?

2. Answer the following questions : 2×5=10

   (a) Prove that section of a sphere by a plane is a circle.

   (b) Discuss the effect of refraction on sunrise.

   (c) Drawing a neat diagram, discuss how horizontal coordinates of a heavenly body are measured.

   (d) Prove that the altitude of the celestial pole at any place is equal to the latitude of that place.

   (e) Show that right ascension α and declination δ of the sun is always connected by the equation $\tan\delta = \tan\varepsilon \sin\alpha$, ε being obliquity of the ecliptic.

3. Answer ***any four*** of the following : 5×4=20

   (a) Deduce Kepler's laws from Newton's law of gravitation.

(b) Show that the velocity of a planet in its elliptic orbit is $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$ where $\mu = G(M+m)$ and a is the semi-major axis of the orbit.

(c) If z_1 and z_2 are the zenith distances of a star on the meridian and the prime vertical respectively, prove that

$$\cot\delta = \operatorname{cosec} z_1 \sec z_2 - \cos z_1$$

where δ is the star's declination.

(d) If H be the hour angle of a star of declination δ when its azimuth is A and H' when the azimuth is $(180° + A)$, show that

$$\tan\phi = \frac{\cos\frac{1}{2}(H'+H)}{\cos\frac{1}{2}(H'-H)}$$

(e) In an equilateral spherical triangle ABC, prove that $2\cos\frac{a}{2}\sin\frac{A}{2} = 1$.

(f) If ψ is the angle which a star makes at rising with the horizon, prove that $\cos\psi = \sin\phi\sec\delta$, where the symbols have their usual meanings.

4. Answer ***any four*** questions of the following : 10×4=40

(a) If the colatitude is C, prove that

$$C = x + \cos^{-1}(\cos x \sec y)$$

where $\tan x = \cot\delta\cos H$ and $\sin y = \cos\delta\sin H$, H being the hour angle.

(b) In any spherical triangle ABC, prove that $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$. Also prove that $\frac{\sin(A+B)}{\sin C} = \frac{\cos a + \cos b}{1 + \cos c}$

(c) Define astronomical refraction and state the laws of refraction. Derive the formula for refraction as $R = k\tan\xi$, ξ being the apparent zenith distance of a heavenly body. Mention *one* limitation of this formula.

(d) On account of refraction, the circular disc of the sun appears to be an ellipse. Prove it.

(e) Derive Kepler's equation in the form $M = E - e\sin E$, where M and E are respectively mean anomaly and eccentric anomaly.

Contd.

(f) Assuming the planetary orbits to be circular and coplanar, prove that the sidereal period P and the synodic period S of an inferior planet are related to the earth's periodic time E by

$$\frac{1}{S}=\frac{1}{P}-\frac{1}{E}$$

Calculate the sidereal period (in mean solar days) of a planet whose sidereal period is same as its synodic period.

(g) Prove that, if the fourth and higher powers of e are neglected,

$$E = M + \frac{e\sin M}{1-e\cos M} - \frac{1}{2}\left(\frac{e\sin M}{1-e\cos M}\right)^3$$

is a solution of Kepler's equation in the form.

(h) Derive the expressions to show the effect of refraction in right ascension and declination.

OPTION–C

Paper : MAT–HE–5066

(Programming in C)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Write *any two* special characters that are used in *C*.

(b) Mention *two* data types that are used in *C* language.

(c) For $x=2,\ y=5$, write the output of the *C* function 'pow (x, y)'.

(d) Convert the mathematical expression
$$z = e^x + \log y + \sqrt{1+x}$$
into *C* expression.

(e) Write the utility of clrscr ( ) function.

(f) Write a difference between local variable and global variable.

(g) Write the *C* library function which can evaluate $|x|$.

2. Answer the following questions : 2×4=8

(a) Write the difference between 'assignment' and 'equality'.

(b) How does 'x + +' differ from '+ + x' ?

(c) What is a string constant ? Give an example.

(d) Write *four* relational operators that are used in *C*.

3. Answer ***any three*** parts : 5×3=15

(a) Explain artihmetic and logical operators in *C* with suitable examples.

(b) List three header files that are used in *C*. Also write their utilities. 3+2=5

$A = 5$; $B = 3$

$A = A + B$;

$B = A - B$;

$A = A - B$;

Write the output of *A* and *B* from the above program segment in *C*.

(c) Write a *C* program to find the sum of all odd integers between 1 and *n*.

(d) Write the general form of do-while loop and explain how it works with the help of a suitable example.

(e) Write the utility of 'break' and 'continue' statements with the help of suitable examples.

4. Why are arrays required in *C* programming ? How are one-dimensional arrays declared and inputs given to array ? Explain briefly with example. Write a program to read given *n* numbers and then find the sum of all positive and negative numbers. 2+3+5=10

Or

How are two-dimensional arrays declared ? Write a *C* program to read a 3×3 matrix and print the same as output. Hence write a *C* program to read a 3×3 matrix, print its transpose and write the determinants of both. 1+4+5=10

5. Write a *C* program for each of the following :

(a) To evaluate the function 5

$$f(x) = x^2 + 2x - 10, \; x \geq 0$$
$$= |x|, x < 0$$

(b) To find the biggest of three numbers. 5

Contd.

Or

Explain with example the 'if' statement and nested 'if' statement in *C*. Write a *C* program to find the roots of a quadratic equation $ax^2 + bx + c = 0$, for all possible values of a, b, c. 5+5=10

6. What is the basic difference between 'Library functions' and 'User-defined functions' ? Mention *two* advantages of using 'User-defined functions'. How are such functions declared and called in a program ? Write a *C* program using function to find the biggest of three numbers. 1+2+2+5=10

Or

Write a *C* programme that reads a number, obtains a new number by reversing the digits of the given number, and then determine the gcd of the two numbers. To build the programme, use two functions — one to find gcd and another to reverse the digits. 10