Total number of printed pages–23

3 (Sem–1/CBCS) MAT HG/RC

2021

(Held in 2022)

MATHEMATICS

(Honours Generic/Regular)

For Honours Generic

Attempt either MAT–HG–1016 ***or*** MAT–HG–1026

For Regular

Attempt MAT–RC–1016

The figures in the margin indicate full marks for the questions.

*Answer **either** in English **or** in Assamese.*

OPTION–A

Paper : MAT–HG–1016/MAT–RC–1016

(Calculus)

Full Marks : 80

Time : Three hours

Contd.

1. Answer the following questions : $1\times10=10$
তলৰ প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ

(a) Write the domain and range of the function $f(x)=\cos^{-1}x$.

$f(x)=\cos^{-1}x$ ফলনটোৰ আদিক্ষেত্ৰ আৰু পৰিসৰ লিখা।

(b) Find the value of the function $\sin^2\frac{3\pi}{8}$.

ফলন $\sin^2\frac{3\pi}{8}$ ৰ মান উলিওঁৰা।

(c) Find the value of $\lim\limits_{x\to0}(1+2x)^{\frac{x+3}{x}}$.

$\lim\limits_{x\to0}(1+2x)^{\frac{x+3}{x}}$ ৰ মান উলিওঁৰা।

(d) State whether the statement is true **or** false :

The function $f(x)=\begin{cases}x^2 & , \ x\le0\\ 1-x & , \ x>0\end{cases}$ is continuous at $x=0$.

তলৰ উক্তিটো সঁচা *নে* মিছা লিখা ঃ

ফলন $f(x)=\begin{cases}x^2 & , \ x\le0\\ 1-x & , \ x>0\end{cases}$

$x=0$ বিন্দুত অবিচ্ছিন্ন।

(e) What is the nth derivative of e^{ax+b} ?

e^{ax+b} ৰ n-তম অৱকলজটো কি?

(f) Expand $e^{\sin x}$ in powers of x by using Maclaurin's infinite series.

মেক্‌লৰিন'ৰ অসীম শ্ৰেণীটো ব্যৱহাৰ কৰি $e^{\sin x}$ ক x ৰ ঘাতত প্ৰসাৰ কৰা।

(g) If $f(x)=x(x-1)$, then on what interval the function f is decreasing ?

যদি $f(x)=x(x-1)$ হয়, তেন্তে f ফলনটো কি অন্তৰালত হ্ৰাসমান হব?

(h) State whether the statement is true **or** false :
"Every differentiable function is continuous."
তলৰ উক্তিটো সঁচা *নে* মিছা লিখা ঃ
"প্ৰত্যেক অৱকলন ফলন অবিচ্ছিন্ন।"

(i) Given a function U that satisfies $1-\frac{x^2}{4} \le U(x) \le 1+\frac{x^2}{2}$ for all $x \neq 0$, find $\lim_{x \to 0} U(x)$.

ফলন U এনে ধৰণে দিয়া আছে, যাতে $1-\frac{x^2}{4} \le U(x) \le 1+\frac{x^2}{2}$ য'ত সকলো $x \neq 0$ তেন্তে $\lim_{x \to 0} U(x)$ উলিওঁৰা।

(j) State whether the statement is true **or** false :

The slope of the tangent line to the curve $y = x^2 + 4x + 7$ at $x = 1$ is 6.

তলৰ উক্তিটো সচাঁ *নে* মিছা লিখা ঃ

$x = 1$ ত $y = x^2 + 4x + 7$ বক্ৰৰ স্পৰ্শকৰ প্ৰৱণতা 6।

2. Answer the following questions : 2×5=10

তলৰ প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ

(a) Evaluate $\cos\left(\text{arc}\cos\left(-\frac{\sqrt{3}}{2}\right)\right)$.

$\cos\left(\text{arc}\cos\left(-\frac{\sqrt{3}}{2}\right)\right)$ ৰ মান উলিওঁৰা।

(b) Let $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

Show that $f(x)$ is continuous at $x = 0$.

ধৰা হল, $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & \text{যদি } x \neq 0 \\ 2 & \text{যদি } x = 0 \end{cases}$

দেখুওঁৰা যে $f(x)$, $x = 0$ বিন্দুত অবিচ্ছিন্ন।

(c) If $y = \sin^{-1} x$, prove that $(1-x^2)y_2 - xy_1 = 0$

যদি $y = \sin^{-1} x$, প্ৰমাণ কৰা যে $(1-x^2)y_2 - xy_1 = 0$

(d) If $u = x^2y + y^2z + z^2x$, then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

যদি $u = x^2y + y^2z + z^2x$, তেন্তে $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ ৰ মান নিৰ্ণয় কৰা।

(e) Evaluate (মান নির্ণয় কৰা) :

$$\lim_{x\to 0} \frac{e^{\sin x}-1}{\log(1+x)}$$

3. Answer ***any four*** questions : 5×4=20
যিকোনো চাৰিটা প্ৰশ্নৰ উত্তৰ দিয়া :

(a) If $y=\left(x^2-1\right)^n$, prove that

$$\left(x^2-1\right)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0.$$

Hence deduce that if $P_n=\frac{d^n}{dx^n}\left(x^2-1\right)^n$,

then $\frac{d}{dx}\left\{\left(1-x^2\right)\frac{dP_n}{dx}\right\}+n(n+1)P_n=0$

3+2=5

যদি $y=\left(x^2-1\right)^n$, প্ৰমাণ কৰা যে

$\left(x^2-1\right)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0$

ইয়াৰ পৰা সাব্যস্ত কৰা যে, যদি

$P_n=\frac{d^n}{dx^n}\left(x^2-1\right)^n$, তেন্তে

$\frac{d}{dx}\left\{\left(1-x^2\right)\frac{dP_n}{dx}\right\}+n(n+1)P_n=0$

(b) State Rolle's theorem and verify it for the function $f(x)=x(x+3)e^{-x/2}$ in $[-3,0]$. 2+3=5

ৰ'লৰ উপপাদ্যটোৰ উক্তি লিখা আৰু সত্যতা f ফলনৰ বাবে $[-3,0]$ অন্তৰালত পৰীক্ষা কৰা য'ত $f(x)=x(x+3)e^{-x/2}$

(c) Evaluate, using L'Hospital's rule :

এল' হ'চপিতাল নীতি প্ৰয়োগ কৰি মান নির্ণয় কৰা :

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$$ 5

(d) Using definition find $\frac{\partial u}{\partial x}$ if $u=\log\left(x^2+y^2\right)$. 5

যদি $u=\log\left(x^2+y^2\right)$, তেন্তে সূত্ৰ প্ৰয়োগ কৰি $\frac{\partial u}{\partial x}$ ৰ মান উলিওঁৰা।

(e) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$. 5

যদি $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$, দেখুওঁৱা যে $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$

(f) Show that the area of triangle ABC is $\frac{1}{2} ab \sin C$. 5

দেখুউৱা যে ABC ত্ৰিভুজৰ ফালি $\frac{1}{2} ab \sin C$

4. Answer **either** [(a) and (b)] **or** [(c) and (d)]: [(a) আৰু (b)] **অথবা** [(c) আৰু (d)] অংশৰ উত্তৰ কৰা ঃ

(a) Find the value of θ in the mean value theorem

$$f(x+h) = f(x) + hf'(x+\theta h),\ 0 < \theta < 1$$

for the function $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$ 5

মধ্যমান উপপাদ্যটোত

$$f(x+h) = f(x) + hf'(x+\theta h),\ 0 < \theta < 1$$

θ ৰ মান উলিওঁৱা য'ত $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$

(b) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$ 5

যদি $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, প্ৰমাণ কৰা যে

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

(c) State Euler's theorem on homogeneous function and then verify for the function

$$f(x, y) = \frac{x\left(x^3 - y^3\right)}{x^3 + y^3}$$ 1+4=5

সুষম ফলনৰ ইউলাৰৰ উপপাদ্যটোৰ উক্তি লিখা আৰু ইয়াৰ সত্যতা f ফলনৰ বাবে পৰীক্ষা কৰা য'ত

$$f(x, y) = \frac{x\left(x^3 - y^3\right)}{x^3 + y^3}$$

(d) Prove that (প্ৰমাণ কৰা) ঃ

$$\frac{x}{1+x^2} < \tan^{-1} x < x, \quad x > 0$$ 5

5. Prove that the double limit exist but repeated limits do not exist for the function

$$f(x, y) = \begin{cases} x \sin\frac{1}{y} + y \sin\frac{1}{x}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$ 10

প্রমাণ কৰা যে ফলন f ৰ দ্বিসীমা স্থিত হয়, কিন্তু পুনৰাবৃত্তী সীমা স্থিত নহয়, য'ত

$$f(x,y)=\begin{cases} x\sin\frac{1}{y}+y\sin\frac{1}{x} & , \ (x,y)\neq(0,0) \\ 0 & , \ (x,y)=(0,0) \end{cases}$$

Or/ অথবা

Find infinite series of $log(1+x)$ for $|x|<1$ using Maclaurin's theorem. 10

মেক্‌লৰিনৰ উপপাদ্যটো ব্যৱহাৰ কৰি $log(1+x)$, $|x|<1$ ৰ অসীম শ্রেণীটো নির্ণয় কৰা।

6. Answer ***either*** [*(a)* and *(b)*] ***or*** [*(c)* and *(d)*]:
[*(a)* আৰু *(b)*] ***অথবা*** [*(c)* আৰু *(d)*] অংশৰ উত্তৰ কৰা ঃ

(a) If (যদি) $y=e^{a\sin^{-1}x}$ prove that (প্রমাণ কৰা যে) —

(i) $(1-x^2)y_2-xy_1-a^2y=0$

(ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$

2+3=5

(b) Show that, $f(x)=x^2\sin\frac{1}{x}$, $x\neq 0$; 0 , $x=0$

is differentiable at $x=0$ and find $f'(0)$.

3+2=5

দেখুউৱা যে, $f(x)=x^2\sin\frac{1}{x}$, $x\neq 0$; 0 , $x=0$

$x=0$ বিন্দুত অৱকলন আৰু $f'(0)$ নির্ণয় কৰা।

(c) A triangle has sides $a=3$ units, $b=4$ units and angle $C=90°$. Find the length of the side c. 5

এটা ত্রিভুজত $a=3$ একক, $b=4$ একক আৰু $\angle C=90°$। c বাহুৰ দৈর্ঘ্যৰ মান উলিওঁৱা।

(d) Evaluate ***any two*** of the following : 2½×2=5

যিকোনো দুটাৰ মান নির্ণয় কৰা ঃ

(i) $\lim\limits_{x\to 0}\frac{\sqrt{x^2+4}-2}{x^2}$

(ii) $\lim\limits_{x\to 0}\frac{1+x+\sin x}{3\cos x}$

(iii) $\lim\limits_{x\to 0}\frac{1-\cos x}{x\sin x}$

(iv) $\lim\limits_{x\to 0}\frac{\tan x\sec 2x}{3x}$

7. Answer **either** [*(a)* and *(b)*] **or** [*(c)* and *(d)*]:
[*(a)* আৰু *(b)*] *অথবা* [*(c)* আৰু *(d)*] অংশৰ উত্তৰ কৰা ঃ

(a) Prove that (প্ৰমাণ কৰা) ঃ

$$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2} = e^{1/3}$$ 6

(b) Show that, $f(x)=\begin{cases}\dfrac{|x-a|}{x-a} & , \ x\neq a\\ 1 & , \ x=a\end{cases}$

is discontinuous at $x=a$. 4

দেখুওৱা যে, $f(x)=\begin{cases}\dfrac{|x-a|}{x-a} & , \ x\neq a\\ 1 & , \ x=a\end{cases}$

$x=a$ বিন্দুত অনবিচ্ছিন্ন।

(c) Show that (*দেখুওৱা*) ঃ

$$\frac{x}{1+x} < \log(1+x) < x \text{ for } x>0$$ 6

(d) Consider the function

$f(x)=x^4-4x^3+6$.

Find the function which

(i) shifts the graph of f two units up;

(ii) shifts the graph of f one unit to the left;

(iii) stretches the graph of f vertically by a factor 3;

(iv) compresses the graph of f horizontally by a factor 2. 4

$f(x)=x^4-4x^3+6$ হ'লে, তলৰ ফলন কেইটা নিৰ্ণয় কৰা ঃ

(i) f ফলনটোৰ লেখটোক ২ একক ওপৰলৈ স্থানান্তৰ কৰা;

(ii) f ফলনটোৰ লেখটোক এক একক বাওঁফালে স্থানান্তৰ কৰা;

(iii) f ফলনটোৰ লেখটোক তিনি গুণ উলম্বভাৱে প্ৰসাৰিত কৰা;

(iv) f ফলনটোৰ লেখটোক দুই গুণ আনুভূমিকভাৱে সংকুচিত কৰা।

OPTION-B

Paper : MAT-HG-1026

(Honours Generic)

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

Answer ***either*** *in English* ***or*** *in Assamese.*

1. Answer the following questions : 1×10=10
তলৰ প্ৰশ্নবোৰৰ উত্তৰ লিখা ঃ

(i) Under what condition

$ax^2 + 2hxy + by^2 = 0$ may represent a pair of parallel straight line ?

কি চৰ্ত সাপেক্ষে $ax^2 + 2hxy + by^2 = 0$ এ এযোৰ পৰস্পৰ সমান্তৰাল সৰলৰেখা নিৰূপণ কৰে?

(ii) Find the point on the conic

$$\frac{8}{r} = 3 - \sqrt{2}\cos\theta$$

whose radius vector is 4.

$\frac{8}{r} = 3 - \sqrt{2}\cos\theta$ শাংকৱৰ ওপৰত থকা বিন্দু এটা নির্ণয় কৰা য'ত ব্যাসার্ধ ভেক্টৰ 4।

(iii) Define conjugate diameters of an ellipse.
উপবৃত্ত এটাৰ সংযুগ্ম ব্যাসৰ সংজ্ঞা দিয়া।

(iv) Express the parabola $y^2 = 4ax$ in parametric form.

$y^2 = 4ax$ অধিবৃত্তটোক প্ৰাচলিক আকাৰত প্ৰকাশ কৰা।

(v) By what angle the axes are to be rotated to remove the xy-term from the equation $ax^2 + 2hxy + by^2 = 0$?
অক্ষ দুডালক কি কোণত ঘুৰালে $ax^2 + 2hxy + by^2 = 0$ সমীকৰণটো xy-পদ মুক্ত হ'ব?

(vi) Define cross product of two vectors.
দুটা ভেক্টৰৰ সদিশ পূৰণৰ সংজ্ঞা দিয়া।

(vii) Find the centre and radius of the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 16$
$(x-1)^2 + (y-2)^2 + (z-3)^2 = 16$
গোলকটোৰ কেন্দ্ৰ আৰু ব্যাসার্ধ নির্ণয় কৰা।

(viii) Find the unit vector that has the same direction as $\bar{u} = 2i + 2j - k$.

$\bar{u} = 2i + 2j - k$ ৰ দিশত এটা ভেক্টৰ নিৰ্ণয় কৰা।

(ix) What is the value of $i \times (i + j + k)$?

$i \times (i + j + k)$ ৰ মান কিমান?

(x) Find parametric equations of the line passing through (4, 2) and parallel to $\bar{v} = (-1, 5)$.

(4, 2) বিন্দুৰ মাজেৰে যোৱা $\bar{v} = (-1, 5)$ ৰ সমান্তৰাল ৰেখাৰ প্ৰাচলিক সমীকৰণ নিৰ্ণয় কৰা।

2. Answer the following questions : 2×5=10

তলৰ প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ

(a) Find the angle between the vectors $\bar{u} = i - 2j + 2k$ and $\bar{v} = -3i + 6j + 2k$.

$\bar{u} = i - 2j + 2k$ আৰু $\bar{v} = -3i + 6j + 2k$ ভেক্টৰ দুটাৰ মাজৰ কোণ নিৰ্ণয় কৰা।

(b) Prove that the equation

$$2x^2 + 3xy - 2y^2 + 7x - y + 3 = 0$$

represents a pair of perpendicular lines.

প্ৰমাণ কৰা যে

$2x^2 + 3xy - 2y^2 + 7x - y + 3 = 0$ এ এযোৰ পৰস্পৰ লম্বভাৱে থকা সৰলৰেখা নিৰূপণ কৰে।

(c) Find the co-ordinates of the focus and the vertex of the parabola

$y^2 - 4y - 2x - 8 = 0$.

$y^2 - 4y - 2x - 8 = 0$ অধিবৃত্তটোৰ কেন্দ্ৰ আৰু শীৰ্ষবিন্দু নিৰ্ণয় কৰা।

(d) Show that if the polar of P w.r. to an ellipse passes through Q, then the polar of Q passes through P.

দেখুওৱা যে এটা উপবৃত্ত সাপেক্ষে P বিন্দুৰ ধ্ৰুৱীয় ৰেখাডাল Q বিন্দুৰেদি গ'লে, Q বিন্দুৰ ধ্ৰুৱীয় ৰেখাডালো P য়েদি যাব।

(e) Find the direction cosines of the vector $\bar{v} = 2i - 4j + 4k$.

$\bar{v} = 2i - 4j + 4k$ ভেক্টৰটোৰ দিশাংক নিৰ্ণয় কৰা।

3. Answer the following questions : ***(any four)*** 5×4=20

তলৰ প্ৰশ্নবোৰৰ উত্তৰ দিয়া ঃ *(যিকোনো চাৰিটা)*

(a) By a suitable transformation remove the term containing xy from the equation $11x^2 + 4xy + 14y^2 = 5$.

এটা যথোপযুক্ত ৰূপান্তৰৰ সহায়ত $11x^2+4xy+14y^2=5$ সমীকৰণটোৰ পৰা xy পদটো বিলোপ কৰা।

(b) Find a vector that is orthogonal to both of the vectors $\vec{u}=(2,-1,3)$ and $\vec{v}=(-7,2,-1)$.

$\vec{u}=(2,-1,3)$ আৰু $\vec{v}=(-7,2,-1)$ ভেক্টৰ দুটাৰ উভয়ৰে লম্ব হোৱা ভেক্টৰ এটা নির্ণয় কৰা।

(c) The normal at the point $(at_1^2, 2at_1)$ meets the parabola again at the point $(at_2^2, 2at_2)$. Prove that $t_2=-t_1-\frac{2}{t_1}$.

$(at_1^2, 2at_1)$ বিন্দুত টনা অভিলম্ব ডালে অধিবৃত্তটোক $(at_2^2, 2at_2)$ বিন্দুত সংযোজিত হয়। প্ৰমাণ কৰা যে, $t_2=-t_1-\frac{2}{t_1}$

(d) Find the polar equation of a conic in the form $\frac{l}{r}=1+e\cos\theta$.

এটা শাংকৱৰ ধ্ৰুৱীয় সমীকৰণ $\frac{l}{r}=1+e\cos\theta$ আকাৰত নির্ণয় কৰা।

(e) Reduce the equation $x^2+4xy+y^2-2x+2y+6=0$ to standard form.

$x^2+4xy+y^2-2x+2y+6=0$ সমীকৰণটোক আদর্শ আকাৰলৈ ৰূপান্তৰ কৰা।

(f) Find the equation of the bisectors of the angles between the pair of lines given by $ax^2+2hxy+by^2=0$.

$ax^2+2hxy+by^2=0$ সমীকৰণে বুজোৱা ৰেখা দুডালৰ মাজৰ কোণৰ সমদ্বিখণ্ডকৰ সমীকৰণ নির্ণয় কৰা।

Answer ***either*** *(a)* ***or*** *(b)* from each of the following ***four*** equations : 10×4=40

তলৰ ***চাৰিটা*** সমীকৰণৰ প্ৰতিটোৰ *(a)* ***অথবা*** *(b)* অংশৰ উত্তৰ কৰা ঃ

4. *(a)* *(i)* Find the equation of the tangent to the conic $4x^2+3xy+2y^2-3x+5y+7=0$ at the point $(1,-2)$.

$(1,-2)$ বিন্দুত $4x^2+3xy+2y^2-3x+5y+7=0$ শাংকৱৰ ওপৰত টনা স্পৰ্শকৰ সমীকৰণ নির্ণয় কৰা।

(ii) Show that a homogeneous equation of second degree in x and y represents a pair of straight lines passing through the origin.
দেখুওৱা যে, x আৰু y ৰ দ্বিঘাতৰ এটা সুষম সমীকৰণে মূলবিন্দুৰ মাজেৰে যোৱা এযোৰ সৰলৰেখাক বুজাব?

(b) If by a transformation from one set of rectangular axes to another with the same origin the expression

$$ax^2 + 2hxy + by^2$$

changes to $ax'^2 + 2hx'y' + by'^2$, then

$a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.
মূলবিন্দু সাপেক্ষে আয়তীয় অক্ষদ্বয়ক ঘুৰালে যদি

$ax^2 + 2hxy + by^2$ ৰাশিটো

$ax'^2 + 2hx'y' + by'^2$ লৈ পৰিবৰ্তিত হ'য়,
তেন্তে দেখুওৱা যে

$a + b = a' + b'$ আৰু $ab - h^2 = a'b' - h'^2$।

5. (a) (i) Find an equation of the line in 3-space that passes through the points $P(2, 4, -1)$ and $Q(5, 0, 7)$.
$P(2, 4, -1)$ আৰু $Q(5, 0, 7)$ বিন্দুৰ মাজেৰে যোৱা ৰেখাৰ সমীকৰণ নির্ণয় কৰা।

(ii) Calculate the scalar triple product $\vec{u}.(\vec{v} \times \vec{w})$ of the vectors $\vec{u} = 3i - 2j - 5k$, $\vec{v} = i + 4j - 4k$ and $\vec{w} = 3j + 2k$.
$\vec{u} = 3i - 2j - 5k$, $\vec{v} = i + 4j - 4k$ আৰু $\vec{w} = 3j + 2k$ হ'লে, $\vec{u}.(\vec{v} \times \vec{w})$ নির্ণয় কৰা।

(b) (i) Find the area of the triangle that is determined by the points $P(2, 2, 0)$, $Q(-1, 0, 2)$ and $R(0, 4, 3)$.
$P(2, 2, 0)$, $Q(-1, 0, 2)$ আৰু $R(0, 4, 3)$ বিন্দুৰে নির্দেশ কৰা ত্রিভুজটোৰ কালি নির্ণয় কৰা।

(ii) Prove that
$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$
প্ৰমাণ কৰা যে,
$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

6. (a) Show that the ortho-centre of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$ is given by $\frac{x}{l} = \frac{y}{m} = \frac{a + b}{am^2 - 2hlm + bm^2}$.

দেখুওৱা যে $ax^2 + 2hxy + by^2 = 0$ আৰু $lx + my = 1$ ৰেখাৰে আবৃত ত্ৰিভুজটোৰ পাদিক কেন্দ্ৰ হ'ল $\frac{x}{l} = \frac{y}{m} = \frac{a+b}{am^2 - 2hlm + bm^2}$

(b) Discuss the nature of the conic represented by

$3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$.

$3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$ সমীকৰণে নির্দেশ কৰা শাংকৱৰ প্ৰকৃতি নির্ণয় কৰা।

7. (a) (i) Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point α is

$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha)$

দেখুওৱা যে $\frac{l}{r} = 1 + e\cos\theta$ শাংকৱটোৰ α বিন্দুত স্পৰ্শকৰ সমীকৰণ হ'ল

$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha)$

(ii) Obtain the equation of the chord of the conic $\frac{l}{r} = 1 + e\cos\theta$, joining the two points on the conic, whose vectorial angles are $(\alpha + \beta)$ and $(\alpha - \beta)$.

$\frac{l}{r} = 1 + e\cos\theta$ শাংকৱটোৰ $(\alpha + \beta)$ আৰু $(\alpha - \beta)$ বিন্দু সংযোগী জ্যাৰ সমীকৰণ উলিওৱা।

(b) (i) Find the condition that the line $a\cos\theta + b\sin\theta = \frac{l}{r}$ may be a normal to the conic $\frac{l}{r} = 1 + e\cos\theta$.

$a\cos\theta + b\sin\theta = \frac{l}{r}$ ৰেখাডাল $\frac{l}{r} = 1 + e\cos\theta$ শাংকৱৰ অভিলম্ব হোৱাৰ চর্ত্ত উলিওৱা।

(ii) Find the polar equation of a circle.
এটা বৃত্তৰ ধ্ৰুৱীয় সমীকৰণ নির্ণয় কৰা।

Total number of printed pages–11

3 (Sem–1/CBCS) MAT HC 2

2021

(Held in 2022)

MATHEMATICS

(Honours)

Paper : MAT–HC–1026

(Algebra)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 10 = 10$

   (a) Find the polar representation of $z = 2i$.

   (b) If $x = 0$ and $y > 0$, then what is the value of t^* ?

   (c) Write the negation of the statement 'For any integer n, $n^2 > n$' in plain English then formulate the negation using set of context and quantifier.

Contd.

(d) Disapprove the statement using counter example :

"For any $x, y \in \mathbb{R}$, $x^2 = y^2$ implies $x = y$."

(e) Suppose f is a constant function from X to Y. The inverse image of a subset of Y cannot be

(i) an empty set

(ii) the whole set X

(iii) a non-empty proper subset of X

(Choose the correct option)

(f) Let $X = Y = \mathbb{R}$. Let $A \subseteq X, B \subseteq Y$. Draw the picture for $A \times B$ where $A = [-1, 1]$ and $B = [2, 3]$.

(g) Suppose a system of linear equations in echelon form has a 3×5 augmented matrix whose fifth column is a pivot column.

Is the system consistent? Justify.

(h) If a set $S = \{\vec{v}_1, \vec{v}_2, ..., \vec{v}_p\}$ in $\mathbb{R}^n$ contains the $\vec{O}$ vector, is the set linearly independent? Justify.

(i) If $A = \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, compute $(A\vec{x})^T$.

(j) What is the determinant of an $n \times n$ elementary matrix E that has been scaled by 7.

2. Answer the following questions : 2×5=10

(a) If $z = -2\sqrt{3} - 2i$, find the polar radius and polar argument of z.

(b) Is the function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = |x - 2|$ one-one and onto? Explain.

(c) Let universal set be $\mathbb{R}$ and index set be $\mathbb{N}$. For a natural number n, $J_n = \left(0, \frac{1}{n}\right)$.

Identify with justification $\bigcap_{n \in N} J_n$.

(d) Show that T is a linear transformation by finding a matrix that implements the mapping

$$T(x_1, x_2, x_3, x_4) = (0, x_1 + x_2, x_2 + x_3, x_3 + x_4)$$

(e) A is a 2×4 matrix with two pivot positions. Answer the following with justification :

(i) Does $A\vec{x} = \vec{0}$ have a non-trivial solution ?

(ii) Does $A\vec{x} = \vec{b}$ have at least one solution for every $\vec{b}$?

3. Answer ***any four*** questions from the following : 5×4=20

(a) Find the polar representation of the complex number

$z = 1 - \cos a + i \sin a \quad a \in [0, 2\pi)$. 5

(b) Let A and B be subsets of an universal set U. Prove —

(i) $(A \cap B)^C = A^C \cup B^C$

(ii) $(A \cup B)^C = A^C \cap B^C$ 5

(c) Define bijection.

Let $f : \mathbb{N} \to \mathbb{N}$ be $f(m) = m - 1$, if m is even $f(m) = m + 1$, if m is odd. Show f is a bijection and $f^{-1} = f$. 1+4=5

(d) For vectors $\vec{v}_1, \vec{v}_2, ..., \vec{v}_p \in \mathbb{R}^n$ define span $\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_p\}$ construct a 3×3 matrix A with non-zero elements and a vector $\vec{b}$ on $\mathbb{R}^3$ such that $\vec{b}$ is not in the set spanned by the columns of A. 2+3=5

(e) Alka-Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet is dissolved in water the following reaction produces sodium citrate, water and carbon dioxide :

$$NaHCO_3 + H_3C_6H_5O_7 \to Na_3C_6H_5O_7 + H_2O + CO_2$$

Balance the chemical equation using vector equation approach. 5

(f) Prove that an $n \times n$ matrix A is invertible if and only if A is row equivalent to I_n, and in this case any sequence of elementary row operations that reduces A to I_n also transforms I_n into A^{-1}. 5

4. Answer ***any four*** from the following : 10×4=40

(a) (i) Find the cube roots of the number $z = 1 + i$ and represent them in the complex plane. 5

(ii) Find the number of ordered pairs (a, b) of real numbers such that $(a+ib)^{2002} = a - ib$. 2

(iii) If x, y, z be real numbers such that $\sin x + \sin y + \sin z = 0$ and $\cos x + \cos y + \cos z = 0$, prove that $\sin 2x + \sin 2y + \sin 2z = 0$ and $\cos 2x + \cos 2y + \cos 2z = 0$. 3

(b) (i) Solve the equation

$$z^7 - 2iz^4 - iz^3 - 2 = 0.$$ 5

(ii) Find the inverse of the matrix if it exists by performing suitable row operations on the augmented matrix $[A : I]$

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$ 5

(c) (i) If $f : X \to Y$ be a map and $B \subseteq Y$, then prove $f^{-1}(B^C) = (f^{-1}(B))^C$. 4

(ii) $A_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$, where $n \in \mathbb{N}$. Find $\bigcup_{n \in N} A_n$ and $\bigcap_{n \in N} A_n$. 2

(iii) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^2$.

Find $f^{-1}(1), f^{-1}(-1), f^{-1}([0, 1])$. 4

(d) (i) State the induction principle and use it to show that for any positive integer $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$. 4

(ii) Write as an implication 'square of an even number is divisible by 4'. Then use direct proof to prove it. 3

(iii) Give proof using contrapositive

'For an integer x if $x^2 - 6x + 5$ is even, then x is odd'. 3

(e) (i) Use the invertible matrix theorem to decide if A is invertible

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 2 \\ -5 & -1 & 9 \end{bmatrix}$$ 2

(ii) Compute *det A* where

$$A = \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$$ 4

(iii) What do you mean by equivalence class for an equivalence relation?

For the relation $a \equiv b \bmod(5)$ on z, find all the distinct equivalence classes of z. 1+3=4

(f) (i) Solve the system of equations

$$x_1 - 3x_3 = 8$$
$$2x_1 + 2x_2 + 9x_3 = 7$$
$$x_2 + 5x_3 = -2$$

4

(ii) Choose h and k such that the system has 4

(a) no solution

(b) a unique solution

(c) many solutions

$$x_1 + hx_2 = 2$$
$$4x_1 + 8x_2 = k$$

(iii) Write the general solution of $10x_1 - 3x_2 - 2x_3 = 7$ in parametric vector form. 2

(g) (i) Prove that the indexed set $S = \{\bar{v}_1, \bar{v}_2, ..., \bar{v}_p\}$ of two or more vectors is linearly dependent if and only if at least one of the vectors in S is a linear combination of the others. In fact, if S is linearly dependent and $\bar{v}_1 \neq \bar{0}$, then some $\bar{v}_j$ (with $j > 1$) is a linear combination of the preceding vectors $\bar{v}_1, \bar{v}_2, ..., \bar{v}_{j-1}$. 5

Contd.

(ii) Use Cramer's rule to compute the solutions of the system 3

$$-5x_1 + 3x_2 = 9$$
$$3x_1 - x_2 = -5$$

(iii) Suppose $T : \mathbb{R}^5 \to \mathbb{R}^2$ and $T(\vec{x}) = A\vec{x}$ for some matrix A and each $\vec{x}$ in $\mathbb{R}^5$.

How many rows and columns does A have ? Justify. 2

(h) (i) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates each point in $\mathbb{R}^2$ about the origin through an angle ϕ with the counter-clockwise direction taken as positive. Find the standard matrix for this transformation. 3

(ii) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation.

Prove that T is one-to-one if and only if the equation $T(\vec{x}) = \vec{0}$ has only the trivial solution. 4

(iii) Find the area of the parallelogram whose vertices are $(0, -2)$, $(6, -1)$, $(-3, 1)$ and $(3, 2)$. 3
