

Important practice question

1. Short Answer Questions ($1 \times 5 = 5$ marks)

- (a) For $n > 1$, the sum of all distinct n th roots of unity is _____.
- (b) If an algebraic equation is of degree n , which of the following statements is always true?
 - (i) It can have at most $n - 1$ roots
 - (ii) It has exactly n distinct real roots
 - (iii) It has n roots in total, real or complex
 - (iv) It can have infinitely many roots
- (c) For a polynomial with real coefficients, the number of positive real roots is equal to the number of sign changes or less by a multiple of _____.
- (d) State whether the following statement is true or false: For any two 2×2 matrices A and B , $(A + B)^2 = A^2 + 2AB + B^2$ holds always.
- (e) A system of m linear equations in n unknowns is called inconsistent if it has:
 - (i) Exactly a single root satisfying all equations
 - (ii) No common root that satisfies every equation
 - (iii) Multiple roots satisfying all equations simultaneously
 - (iv) None of the above

2. Answer any six questions ($2 \times 6 = 12$ marks)

- (a) Using Descartes' Rule of Signs, determine the possible number of real and complex roots of the equation $x^4 - 3x^3 + 3x^2 - x + 2 = 0$.
- (b) If α and β are the roots of the equation $ax^2 - bx + c = 0$, express the sum and product of the roots in terms of the coefficients.
- (c) Define symmetric functions of the roots of an equation. Give one example.
- (d) If A is a square matrix such that $A^T = -A$, what special property does A have? Explain briefly.
- (e) If a square matrix A satisfies $A^2 = 0$ (the zero matrix), can A^{-1} exist? Justify your answer.
- (f) When does a homogeneous system of linear equations have only the trivial solution, and when does it have infinitely many solutions?

3. Section 3 ($5 \times 4 = 20$ marks)

- (a) Let $n > 1$ be a positive integer. Find all real n th roots of 1 for the cases when n is even and when n is odd.
- (b) If the roots of the quadratic equation $x^2 - 5x + 6 = 0$ are transformed such that each root is increased by 3, form the new quadratic equation whose roots are the transformed roots.
- (c) Explain whether the sets of all symmetric and all skew-symmetric matrices are each closed under matrix addition.
- (d) Find the reduced row echelon form of the following matrix and determine its rank:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 1 & 1 & 2 & 3 \end{pmatrix}$$

4. Long Answer Questions ($5 \times 7 = 35$ marks)

- (a) For the cubic equation $x^3 - 4x^2 + px - q = 0$, two of its roots are 1 and $(p - 2)$. Using only the relations between roots and coefficients, find the values of p and q .
- (b) Solve by Cardan's method: $x^3 - 6x^2 + 11x - 6 = 0$.
- (c) If $AB = I$ for square matrices A and B of the same order, prove that $B = A^{-1}$.
- (d) Prove that the following statements for a square matrix A are equivalent:
- A is nonsingular.
 - The equation $Ax = 0$ has only the trivial solution.
 - $\det(A) \neq 0$.
- (e) Explain briefly how the augmented matrix of a linear system of equations helps in determining whether the system is consistent.
- (f) Test the consistency of the following system of equations and solve if it is consistent:

$$\begin{aligned} x + y + z &= 6 \\ x + 3y + z &= 10 \\ x + 2y + 2z &= 8 \end{aligned}$$

- (g) For a homogeneous system $Ax = 0$, explain how the number of free variables in the reduced matrix relates to the number of possible solutions.

(h) Determine all solutions of the following homogeneous system:

$$x + 2y - z = 0$$

$$2x + 5y - 3z = 0$$

$$3x + 8y - 5z = 0$$