FYGUP 1st semester

Paper - classical Algebra

Important question

(1) If $x+\frac{1}{x}=2\cos\theta$ prove that

$$x^n+\frac{1}{x^n}=2\cos n\theta$$

(2) If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$ prove that

(i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$

(ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$

(3) If $\tan\log(x+iy)=a+ib$ if $a^2+b^2\neq1$ then prove that

$$\tan\log(x^2+y^2)=\frac{2a}{1-a^2-b^2}$$

④ ~~Q~~ Solve the equation
$x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ given
that one root is $2+\sqrt{3}$

⑤ If $1, \alpha, \beta, \gamma, \delta$ are the ~~roots~~
zeroes of $x^5 - 1$, then prove that
$(1-\alpha)(1-\beta)(1-\gamma)(1-\delta) = 5$
$\alpha, \beta, \gamma, \delta$ are distinct from 1

⑥ Find the equation whose
roots are the squares of
the roots of the equation
$x^3 + qx + r = 0$

⑦ Use cardon's method to
solve $28x^3 - 9x^2 + 1 = 0$

⑧ If $A = \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
find scalar K so that
$A^2 + I = KA$

(9) Find the rank of the matrix
$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$$

(10) show that the only real value of λ for which the equation
$$x+2y+z=\lambda x \quad x+y+2z=\lambda y$$
$$2x+y+z=\lambda z$$
have a non-trivial solution is 4. Also solve the equation for $\lambda=4$