



জৱাহরলাল নেহরু মহাবিদ্যালয়
JAWAHARLAL NEHRU COLLEGE

A Project Report

On

***Fibonacci Numbers: Its History, manifestation in
nature and application in financial market analysis.***

Master of Science in Mathematics



Submitted By-

Keshab Kanti Saha

G.U.ROLL NO: PS-221-019-0007

G.U.Reg No: 19012501

Session: 2023-24

Submitted to

Department of Mathematics

Jawaharlal Nehru College, Boko

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my project guide, **Sir Ripunjoy Choudhury**, Department of Mathematics, Jawaharlal Nehru College, Boko for his constant support, inspiration, cordial advice and invaluable guidance through-out the course of this project.

I find myself spellbound to acknowledge **Dr. Dipankar Sarma**, H.O.D. (Mathematics), Jawaharlal Nehru College, Boko for providing the required facilities from the Department and all kind of academic help needed during the course of this project.

I would also like to extend my thankful note to all the Faculty members of Department of Mathematics, Jawaharlal Nehru College, Boko and my fellow batchmates. Last but not the least I am indebted to my family for their support, patience and understanding.

Date: 26/12/2023

Keshab Kanti Saha

Keshab Kanti Saha

(PS-221-019-0007)



DECLARATION

I hereby declare that this project report has been prepared by me during the year 2023-24 under the guidance of **Sir Ripunjoy Choudhury** in the partial fulfillment of the syllabus for 3rd semester of M.Sc (Mathematics) degree.

I also declare that this project report represents largely my own understandings and words in this topic. This report has been prepared without resorting to plagiarism and adhered to all principles of academic honesty and integrity. No falsified or fabricated data or information has been presented.

Finally I declare that this project report is the outcome of my own efforts and that it has not been submitted elsewhere for the award of any degree.

.....*Keshab Kanti Saha*.....

Keshab Kanti Saha

(PS-221-019-0007)

G.U.Reg No: 19012501

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Abstract

In this project, Fibonacci sequences is integrated into the field of stock market analysis, a mathematical phenomenon. The study investigates the inherent patterns within Fibonacci sequences and their potential relevance in predicting stock market trends. By applying Fibonacci retracement levels and extensions, the project aims to uncover meaningful correlations between Fibonacci ratios and key market indicators. This project provides insights that may enhance forecasting accuracy and risk management strategies in the dynamic realm of stock market.

Objective

The main aim of this project is to study and understand the Fibonacci number, the sequence and to develop a comprehensive study that combines Fibonacci sequence analysis and stock market analysis to explore potential correlations, leveraging Fibonacci retracement and extension levels to enhance technical analysis and make informed investment decisions.

Introduction

1

The Fibonacci Sequence, an intriguing musical work of numbers, has captivated mathematicians and scientists as well as fans since its discovery by Leonardo Pisano (Fibonacci) in 1202. In this project we will set out to explore the elegant depths of this delightful series and its mathematical prerequisites, utilities as well as inter-relationship with nature and mainly its importance in the stock market analysis.

At its core, the Fibonacci Sequence is a sequence of numbers where each term is the sum of the two preceding ones: 0, 1, 1, 2, 3, 5, ... and thereafter. This cyclical pattern extends infinitely, and exhibits a particular harmony and organisation that is quite different from simple numbering.

Perhaps the most fascinating fact of the Fibonacci Sequence is its appearance in nature. This project sets out to explore the beautiful manifestations of a wisp in nature, from the spiral whorls that decorate seashells right through to leaves on a stem. Through the study of actual cases, we hope to demonstrate that mathematical concepts are connected with our own living beauty.

We take our examination of the Fibonacci sequence beyond² pure mathematics, considering their many applications and far-reaching implications. From number theory to the golden ratio, let this project illuminate a few of that all-around useful mathematical weapon's many applications. Most of all, we investigate its applications in stock market analysis.

The significance of this Fibonacci sequence both in theory and application. This exploration serves as a stepping stone for future investigations, inspiring curiosity about the sequence's potential applications in emerging fields and its role in shaping our understanding of complex systems. Our journey through the Fibonacci sequence is not just an exploration of numbers; it is an odyssey into the heart of mathematical beauty.

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History

3

Leonardo of Pisa, or Fibonacci, was born in Pisa, Italy around 1170. He was an Italian mathematician. In his role as the introducer of the Hindu-Arabic numeral system to the Western world, he is probably most well-known for the Fibonacci Sequence, the series of numbers where each number is the sum of the two preceding ones (0, 1, 1, 2, 3, 5, 8, 13, 21, and so). He introduced this sequence to the Western world in his 1202 book "Liber abaci", where he posed and solved a problem involving the growth of a population of rabbits. However, the sequence had been previously described in Indian mathematics, specifically in the work of Indian mathematicians such as Virahanka and Gopala, who were active between the 6th and 12th centuries. Their contributions are found in various mathematical texts, including the "Pingala - Sutras".

In the context of Pingala's work, the Fibonacci Sequence appears as a solution to a problem related to poetic structures known as meters. Pingala considered patterns of syllable combinations in poetic

verses and introduced a recursive algorithm to count ⁴ the number of such combinations. This algorithm gives rise to a sequence of numbers that is essentially a form of the Fibonacci sequence. Though not presented in the familiar format we know today, the underlying principle of Recursive addition is evident in Pingala's work.

One concept of the Fibonacci sequence is the introduction of Golden ratio. The golden ratio, rooted in ancient Greece with Euclid, emerged as a geometric and aesthetic principle. In the 20th Century, the golden ratio's relation in mathematics, arts and design were seen to come forward.

In near recent times, a concept has been introduced where Fibonacci sequence is used to find retracement levels and resistance to stocks and related analysis as well. Basically, traders use these Fibonacci numbers in identifying key levels and patterns in the financial market.

The Fibonacci Sequence.

5

In the Liber Abaci, Fibonacci posed the following problem dealing with the number of offspring generated by a pair of rabbits conjured up in the imagination:

A man put one pair of rabbits in a certain place entirely surrounded by a wall. How many pairs of rabbits can be produced from that pair in a year, if the nature of these rabbits is such that every month each pair bears a new pair which from the second month on becomes productive?

If none of the rabbits die, a pair is born in the first month. Thus two pairs of them are present. By the second month the original couple has produced another pair. One month later, both the original pair and the firstborn pair have produced another new pair, so that three adult and two young pairs are present, and so on. Likewise, each month the young pairs grow up and become adult pairs, making the new "adult" entry the previous one plus the previous "young" entry.

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Each of the pairs that was adult last month produces one young pair, so that the new "young" entry is equal to the previous "adult" entry.

If continued indefinitely, the sequence encountered in the rabbit problem 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... is called the Fibonacci sequence and its terms the Fibonacci numbers.

Growth of rabbit colony

Months	Adult pairs	Young pairs	Total
1	1	1	2
2	2	1	3
3	3	2	5
4	5	3	8
5	8	5	13
6	13	8	21
7	21	13	34
8	34	21	55
9	55	34	89

Now, let the first term of the sequence be u_1 , second term be u_2 , third term of the sequence as u_3 and so on.

Interestingly, the Fibonacci sequence exhibits a property, namely,

$$2 = 1 + 1$$

$$\text{or } u_3 = u_2 + u_1$$

$$3 = 2 + 1$$

$$\text{or } u_4 = u_3 + u_2$$

$$\vdots$$

$$u_n = u_{n-1} + u_{n-2}; \text{ the general formulation technique.}$$

The Golden Ratio

7

The Golden Ratio, denoted by the Greek numeral Phi (ϕ), is an irrational number that represents a proportion that has been lauded as the most perfect and beautiful throughout history. Using a line segment as an example, the Golden Ratio could be attained when, $\frac{a}{b} = \frac{a+b}{a}$ for $a > b$.

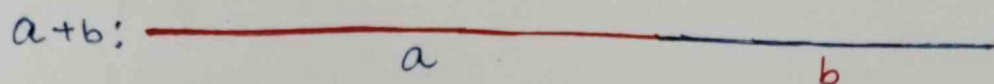


Figure: Golden Ratio Line Segment.

Using this line segment representation and the quadratic formula, ϕ can be derived as:

$$\frac{a+b}{a} = \frac{a}{b} = \phi$$

$$\Rightarrow \frac{a+b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\phi}$$

$$\Rightarrow 1 + \frac{1}{\phi} = \phi$$

$$\Rightarrow \phi^2 - \phi - 1 = 0$$

Thus by the quadratic formula, we have

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

Interestingly, after the reciprocal of phi is simplified, it turns out to be only one less than phi.

$$\frac{1}{\phi} = \frac{b}{a} = \frac{2}{1+\sqrt{5}} = \frac{\sqrt{5}-1}{2} - 1 = \phi - 1 = 0.6180339887...$$

This reveals a very unique relationship between phi and its reciprocal.

Case 1: $\phi - \frac{1}{\phi} = 1$

Case 2: $\phi \cdot \frac{1}{\phi} = 1$.

Which reveals that phi and the reciprocal of phi are the only two numbers whose difference and product are both equal to one.

Golden Ratio can also be found using the Fibonacci numbers.

$$\frac{F_8}{F_7} = \frac{21}{13} = 1.6153846153...$$

However, the larger the number used, the closer the result will be to the actual value of phi.

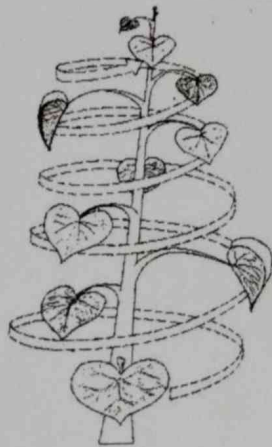
$$\frac{F_{20}}{F_{19}} = \frac{6765}{4181} = 1.6180339632...$$

Conversely, if a Fibonacci number is divided by the following Fibonacci number, the result will be close to the reciprocal of phi. Again, the larger the two numbers used, the closer the result.

Fibonacci's Manifestation in nature.

1) leaves of a plant.

In the dispositions of leaves on plants, the influence of Fibonacci is elegantly manifest. Following the Fibonacci sequence, leaves tend to sprout in a spiral fashion. This arrangement ensures that each leaf gets a fair share of sun, and optimum exposure to sunlight. This angle between successive leaves is the golden angle, a derivative of the golden ratio found in Fibonacci numbers. This packing is space-efficient and promotes effective photosynthesis. The remarkable prevalence of Fibonacci in leaf arrangement gives eloquent testimony to nature's careful application of mathematical principles, showcasing a seamless integration of form and function in the heart of the botanical world.



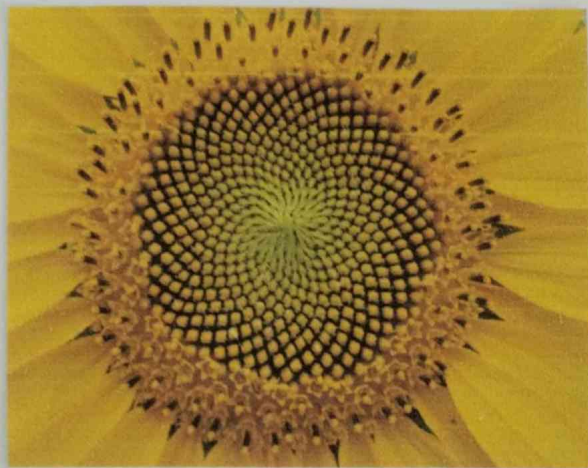
2. Nautilus

The Fibonacci sequence has an intriguing connection with seashells, best represented by the nautilus. When these marine creatures grow, they build their shells in a spiral sequence of chambers. The proportions of these chambers agree with Fibonacci numbers and the golden ratio, making the whole structure aesthetically pleasing as well as efficient. The mathematical precision manifest in the spiral growth of seashells, as guided by the Fibonacci sequence, makes one aware of the fascinating dance between the sciences and nature. For one thing, the inherent patterns make for not only a functional design, but also a harmonious one in the intricate world of marine life.



3. Sunflower

The interaction between sunflower and Fibonacci is a particularly profound one. The seeds' spiral arrangement is based on Fibonacci numbers, and optimizes packaging efficiency. The placement of each seed is determined by the Golden angle, a geometric concept based on the golden ratio derived from the Fibonacci sequence. The close link between sunflowers and the Fibonacci sequence brings out the delicate balance between mathematical concepts and physical reality; no wonder the flower's design is so amazingly vivid and economical.



Stock market analysis using Fibonacci Ratio.

Stock market analysts use different ways in making business forecasts. One such indicator is Fibonacci extensions, which is a tool used to identify potential price targets based on the Fibonacci sequence. By using this Fibonacci extensions, traders can identify potential support and resistance levels that can be used to make informed trading decisions.

The Fibonacci Sequence is :

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, ...

Retracement levels

Type 1: Divide a Fibonacci number by its immediate succeeding number.

$$\frac{89}{144} = 0.618, \frac{144}{233} = 0.618.$$

In General, $\frac{U_{n-1}}{U_n}$, when expressed in percentage for any value of n , where $n \in \mathbb{N}$ yields 61.8%.

Type 2: Divide a Fibonacci number by a number two places higher.

$$\frac{43}{34} = 0.382, \frac{34}{89} = 0.382$$

In General, $\frac{U_{n-2}}{U_n}$, when expressed in percentage for any $n \in \mathbb{N}$ yields 38.2%

Type 3: Dividing a Fibonacci number by a number 3 place higher.

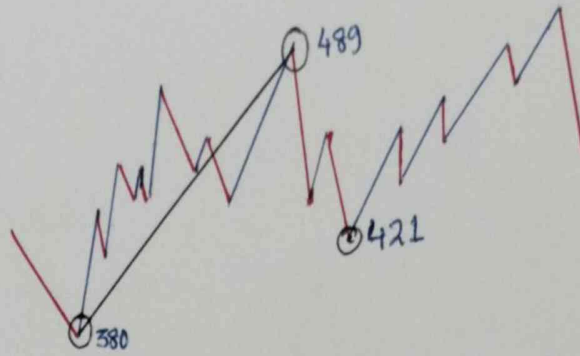
$$\frac{13}{55} = 0.236, \quad \frac{55}{233} = 0.236$$

In General, $\frac{U_{n-3}}{U_n}$ when expressed in percentage using any value for n , where $n \in \mathbb{N}$ yields 23.6%.

Fibonacci Retracement Levels	
1. $\frac{U_{n-1}}{U_n}$	61.8
2. $\frac{U_{n-2}}{U_n}$	38.2
3. $\frac{U_{n-3}}{U_n}$	23.6

It is believed that the Fibonacci ratios, i.e. 61.8%, 38.2%, and 23.6%, finds its application in stock charts. Fibonacci analysis can be applied when there is a noticeable up-move or down-move in prices. Whenever the stock moves either upwards or downwards sharply, it usually tends to retrace back before its next move. 'The retracement level forecast' is a technique that can identify upto which level retracement can happen. The trader can use these levels to position himself for trade.

Chart 1



Here, the two points on the chart, at Rs. 380 where the stock started its rally and at Rs. 489, where the stock prices peaked.

We have a move of 109 points (380 - 489) as the Fibonacci upmove. Now, as per the Fibonacci retracement theory, after the upmove one can anticipate a correction in the stock to last upto the Fibonacci Ratios.

Total Fibonacci up move = 109

23.6 % of Fibonacci up move = $23.6\% \times 109 = 25.72$

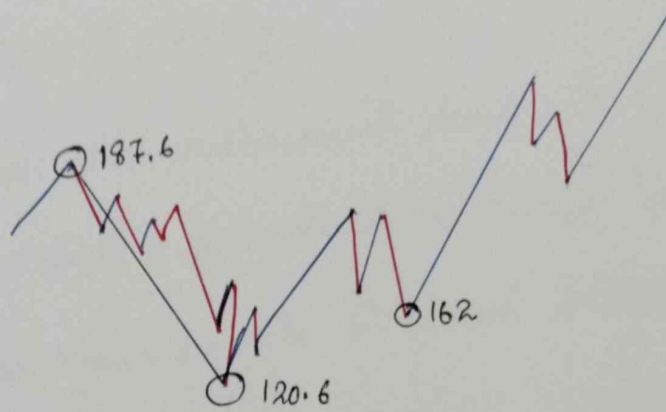
Retracement at 23.6% = $489 - 25.72 = 463.28$

Retracement at 38.2% = $489 - (38.2\% \times 109) = 447.37$

Retracement at 61.8% = $489 - (61.8\% \times 109) = 421.6$

Here, the stock has retraced at 421, i.e. at 61.8% retracement level and continued its up trend.

Chart-2.



Here the stock of DLF limited started to decline from Rs 187.6 to Rs 120.6, thus making 67 points as the Fibonacci down move. Now, according to the principle of the Fibonacci retracement theory, and psychology, we can expect a positive upmove very shortly.

Total Fibonacci down move = 67.

$$\text{Retracement at } 23.6\% = 120.6 + (23.6\% * 67) = 136.41$$

$$\text{Retracement at } 38.2\% = 120.6 + (38.2\% * 67) = 146.19$$

$$\text{Retracement at } 61.8\% = 120.6 + (61.8\% * 67) = 162.006$$

After the down move, stock attempted to bounce back retracing back to Rs.162, which is the 61.8% Fibonacci retracement level.

Golden Zone of Trading

Among all the retracement levels, the 61.8% Fibonacci retracement level is considered a critical level by traders. It is also known as the Fibonacci retracement golden ratio.

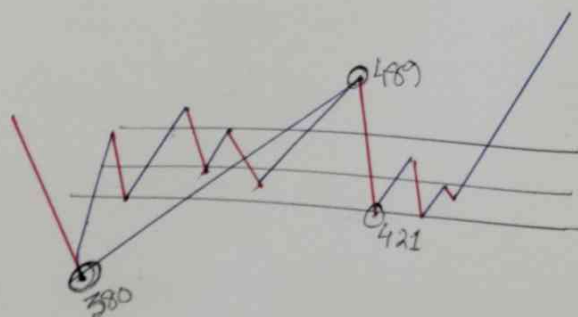
Traders use these areas to anticipate where the price may reverse, providing strategic entry and exit points. The 61.8% and 50% levels form the crucial Golden Zone, which signals potential stabilization or reversal points after significant market movements. Sometimes 38% is also considered in the price-action strategy.

Extended Retracement.

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As all the Fibonacci retracement works best on a larger time-frame, say 10-15 days, day-traders also uses 30-60 minute time frame as well for finding the retracements. we may consider another technique for a shorter time frame.

Chart



If we consider the 23.6% retracement,

$$\text{Retracement at } 23.6\% = 489 - 25.72 = 463.28$$

Now, we consider the 489 as the high point and the new 463 levels as the low point.

Then, we have 26 points ($489 - 463$) as the move.

Now, applying retracement again, for all the levels, particularly of the golden Fibonacci retracement, i.e. 61.8%.

$$\text{Retracement at } 61.8\% = 489 - (61.8\% \text{ of } 26)$$

$$= 489 - 16.06$$

$$= 472.$$

This retracement may also be considered for a shorter time frame, especially for short selling.

Limitations and Suggestions.

One of the main limitations in Fibonacci extensions is that they are based on past price movements and do not take into account the current market conditions. Also, Fibonacci retracement trading fail to account for unexpected market events, such as political crisis and natural disaster. False signals may lead to significant losses, and traders must use other indicators along with the Fibonacci extension signals.

Though Fibonacci retracement gives accurate support and resistance for most of the time, it's still advised to be aware of the limitations of Fibonacci extensions, in placing stop-loss to limit potential losses and protect profits.

Result and Conclusion

19.

In Conclusion, leveraging Fibonacci analysis in stock market trends provides valuable insights into potential support and resistance levels. By applying Fibonacci retracement and extension tools, investors can make more informed decisions, identify key price levels, and enhance their overall trading strategy. The integration of Fibonacci principles adds a systematic and analytical dimension to stock market analysis, aiding traders in navigating market fluctuations with a heightened level of precision.