



জৱাহরলাল নেহরু মহাবিদ্যালয়
JAWAHARLAL NEHRU COLLEGE

A Project Report

On

A study of conformal mapping using complex analysis
("Focusing to solve the forces in wing aerodynamics")

Master of Science in Mathematics



Submitted By

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DECLARATION

I hereby declare that this project report has been prepared by me during the year 2024-25 under the guidance of Sir Ankur Jyoti Kalita in the partial fulfilment of the syllabus for 3rd semester of M.Sc (Mathematics) degree.

I also declare that this project report represents largely my own understandings and words in this topic. This report has been prepared without resorting to plagiarism and adhered to all principles of academic honesty and integrity. No falsified or fabricated data or information has been presented.

Finally I declare that this project report is the outcome of my own efforts and that it has not been submitted elsewhere for the award of any degree.

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CERTIFICATE

This is to certify that the project report as review work entitled as A study of conformal mapping using complex analysis ("Focusing to solve the forces in wing aerodynamics") is a brief introduction on the use of conformal mapping for analysing various shapes and forces of wing aerodynamic system and their efficiency. This project has a real life problem to depict the fact that using conformal mapping make their solution more accurate, which is the outcome of the study and investigation carried out by Nitushree Ray, a student of M.Sc. 3rd semester in the mathematics department of Jawaharlal Nehru College, under my supervision and guidance No part of this project has been submitted for the award of any other degree in other institution.

Nitushree Ray, fulfills the requirement of the regulation related to the nature and prescribed period of the project work for the fulfilment of 3rd semester course syllabus of Gauhati University .

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ABSTRACT

In wing aerodynamics, *conformal mapping* is a mathematical technique that used to transform a complex aerofoil shape into a simpler, more manageable geometric form—often a circle or a circular arc. This transformation makes it easier to analyse the airflow around the wing, because the flow equations (which govern how air moves over a surface) are much simpler to solve for regular shapes like circles.

Once the flow is solved for the simpler shape, the results can then be "mapped back" to the original, more complex aerofoil shape to obtain the actual flow characteristics. Essentially, *conformal mapping* helps to reduce the complexity of aerodynamic analysis by working with a simplified version of the aerofoil geometry, making it easier to predict how the wing will interact with airflow.

KEY POINTS ABOUT CONFORMAL MAPPING IN WING AERODYNAMICS:

- **Complex variable representation:** Aerofoil shapes are represented by using complex number.

$$z=x+iy$$

where the mapping functions transforms the complex coordinates of the aerofoil to a new complex plane often a circle or a straight line.

- **Preserving angles:** The key feature of conformal mapping is that it preserves angles between curves at each point, which is crucial for accurately representing the flow behaviour around an aerofoil.

- **Solving for potential flow:** By applying a conformal mapping to the aerofoil geometry we can solve the complex velocity potential of the flow around the wing, which can then be used to calculate lift, drag and other aerodynamic forces.

INTRODUCTION

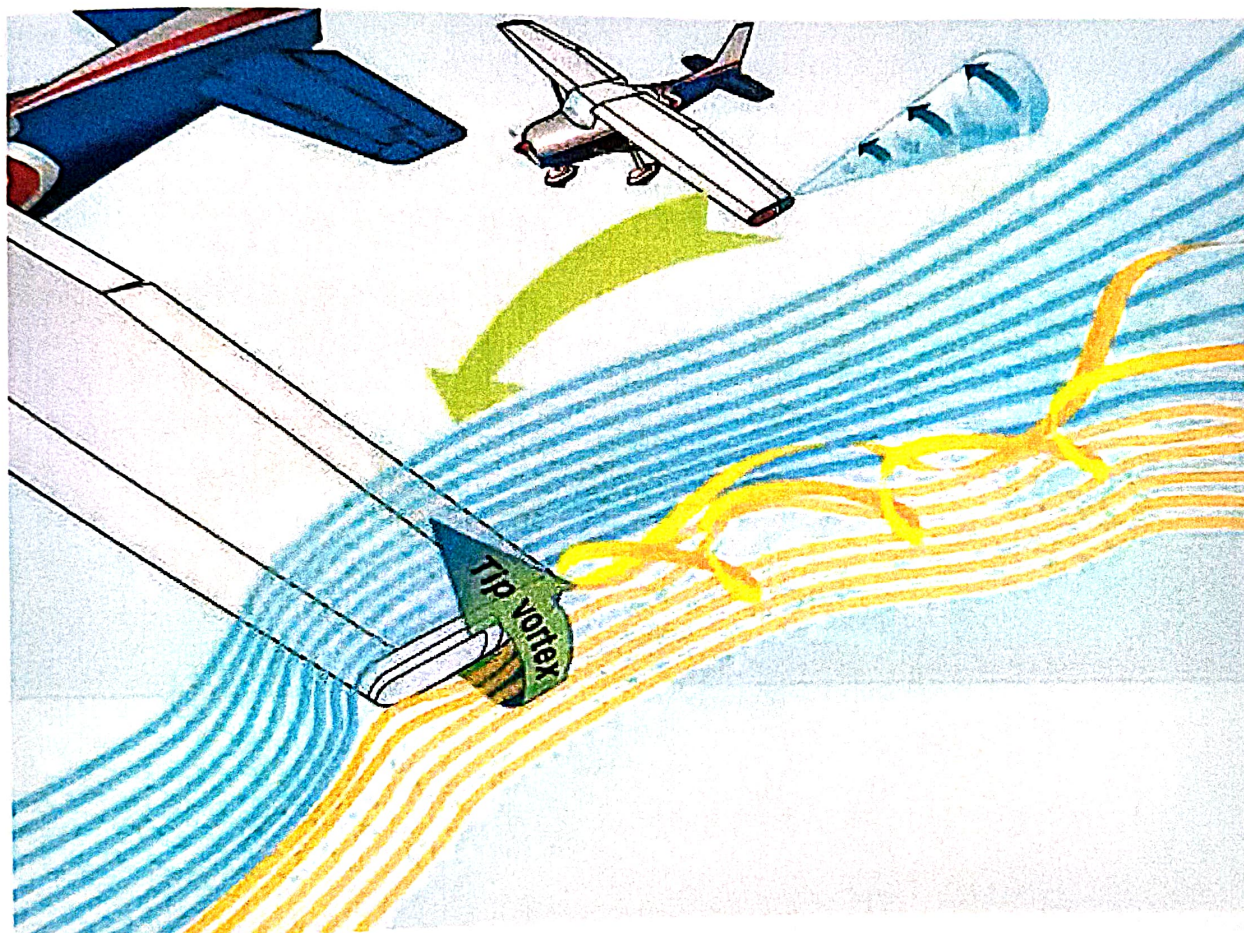
The fundamental problem of wing theory is to understand and predict the flow behaviour around a wing or a wing system, which is geometrically defined. This involves determining how air or other fluids move in relation to the surfaces of a wing as it travels at a constant velocity through the surrounding medium. A crucial aspect of this problem is calculating the pressure distribution along the boundary of the wing, as this influences the lift, drag, and other aerodynamic forces acting on the aircraft.

In the context of fluid mechanics, conformal mapping has played a significant role in addressing this problem. The technique of conformal mapping allows one to transform complex geometries into simpler ones, making it easier to solve the governing equations of fluid flow, particularly the potential flow equations. The use of conformal mappings in fluid mechanics can be traced back to the work of several pioneering mathematicians and physicists, including Carl Friedrich Gauss, Bernhard Riemann, Karl Weierstrass, Carl Neumann, Heinrich Anton de Schwarz, and David Hilbert. These individuals developed mathematical concepts and tools that laid the foundation for understanding complex fluid dynamics.

Lord Rayleigh, a renowned British physicist and mathematician, is often credited with providing the first comprehensive treatment of conformal mapping in the field of aerodynamics. His work contributed greatly to the development of the mathematical tools necessary for understanding the aerodynamics of wings.

The early 20th century saw the emergence of critical contributions from two influential figures: Martin Kutta, a German mathematician, and Nikolay Zhukovsky (often referred to as Joukowski in modern literature), a Russian scientist. Their joint efforts led to the development of aerofoil theory, which became a cornerstone of modern aerodynamics. This theory provided a systematic approach for understanding the aerodynamic properties of aerofoils, specifically the lift generation mechanism, and it marked the beginning of a new era in fluid mechanics and aerodynamics.

Their work, along with the broader contributions of the mathematicians and physicists mentioned, helped establish the theoretical framework that underpins much of the modern understanding of aerodynamics. The combination of mathematical techniques such as conformal mapping and the development of aerofoil theory has been instrumental in advancing the design and performance analysis of wings and other aerodynamic surfaces.



Fluid flow around an aerofoil

AIM OF THE STUDY

The purpose of this exposition is to give an elementary introduction to the use of conformal mapping in two-dimensional aerofoil theory with ideal fluids. Further sections provide us with the prerequisite background knowledge of basic aerofoil theory and two-dimensional fluid dynamics respectively. Application part will show how conformal mappings are used to reconcile the complicated geometries of aerofoils, resulting in a simplification of the problem.

BACKGROUND OF THE STUDY

THEORY OF COMPLEX ANALYSIS:

A. Differentiability and C-R equation:

- I. A complex function f is defined in a non-empty open set D is differentiable at $z_0 \in D$, if the limit exists.

$$\lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} = \lim_{h \rightarrow 0} \frac{f(z_0 + h) - f(z_0)}{h}$$

Here the value of the limit is denoted by $f'(z_0)$ is called the derivative of f at z_0 . The function f is said to be differentiable in D if it is differentiable at every point of D .

- II. A function f is said to be analytic or holomorphic at a point $a \in \mathbb{C}$ if it is differentiable at every point of some neighbourhood of a . Similarly, f is analytic in an arbitrary set S if it is differentiable at every point of some open set containing S . An entire function is the one which is analytic in the whole complex plane and singular point is a point at which a function is said to be analysis.

- III. If $f(z) = u(x,y) + iv(x,y)$ is differentiable at z_0 ,

Then C-R equation hold at $z_0 = x_0 + iy_0$.

If $x(z_0) = f_y(z_0)i$

Or equivalently,

$$u_x(x_0, y_0) = v_y(x_0, y_0) \text{ \& } u_y(x_0, y_0) = -v_x(x_0, y_0).$$

- IV. Let $f: U \longrightarrow \mathbb{C}$, where U is an open set in $\mathbb{C}$ containing the point z_0 , &

$f(z) = u(x,y) + iv(x,y)$ for $z = x + iy \in U$ suppose that;

- a. u_x, u_y, v_x & v_y exist at each point in neighbourhood of z_0 and are continuous at z_0 .

- b. The C-R equation is valid at z_0

Then f is differentiable at z_0 & $f'(z_0) = u_x(z_0) + iv_x(z_0)$

B. Harmonic function:

A real-valued function $\phi = \phi(x, y)$ of real variables x and y is said to be harmonic in an open subset Ω of $\mathbb{C}$ if it has continuous partial derivatives of second order and satisfies the Laplace's equation in two variables $\nabla^2 \phi = 0$ throughout Ω , where ∇^2 is the second order differential operator given by;

$$\nabla^2 \phi = \phi_{xx} + \phi_{yy}$$

The operator $\nabla^2 \phi$ is called **Laplace operator** or simply a **Laplacian**. A function v is called a conjugate harmonic function (or harmonic conjugate) for a harmonic function u in Ω whenever,

$$F = u + iv, \text{ is analytic in } \Omega.$$

Note that the word conjugate here is not the same as conjugate of a complex number z . Further, the harmonic conjugate v is unique, up to an addition of a real constant. Indeed, if v_1 is another harmonic conjugate of u , so that $F = u + iv_1$ is also analytic in Ω .

Then the difference,

$$F - f = i(v_1 - v) \text{ becomes analytic in } \Omega.$$

C. Principle of conformal mapping:

Complex mapping functions-

Let us consider,

a complex variable $z = x + iy$.

Let an analytic function of z i.e. $w = u + iv$ where u and v are real & imaginary parts of w , then the Jacobian is given as

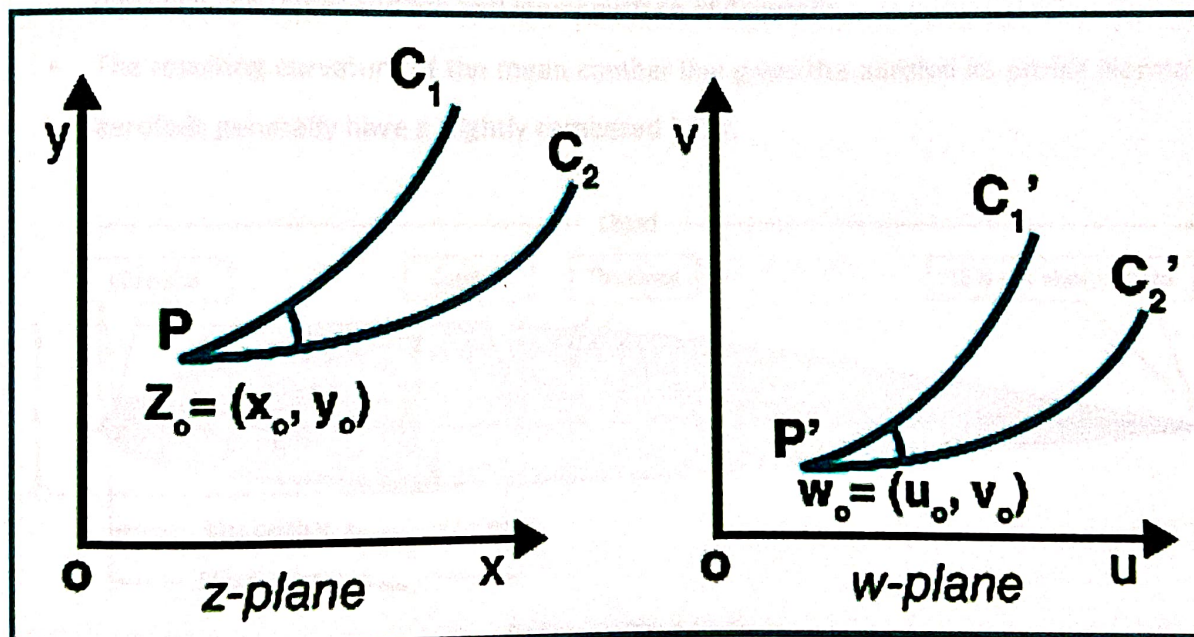
$$\delta(u, v) / \delta(x, y) = |f'(z)|^2$$

It follows that the transformation is one to one in regions where $f'(z) \neq 0$.

CONFORMAL MAPPING

Let the point (x_0, y_0) of the xy plane is mapped into point (u_0, v_0) of the uv plane as shown in the figure while the curves c_1 and c_2 (intersecting at (x_0, y_0)) are mapped respectively into curves c_1' and c_2' (intersecting at (u_0, v_0)). Then the transformation is such that the angle at (x_0, y_0) between c_1 and c_2 is equal to the angle at (u_0, v_0) between c_1' and c_2' both in magnitude and sense, then the transformation is said to be conformant at (x_0, y_0)

A mapping that preserves the magnitude of angle but not necessarily the sense is called isogonal.



BASIC AEROFOIL THEORY AND TERMINOLOGY:

The objective of wing theory is to investigate the aerodynamic action on a wing or system of wings. The embedding on the wing with given velocity is described by Bernoulli's principle.

An aerofoil in our context is the shape of a wing as seen in cross-section.

- The **chord line** means the direct line joining the leading edge of aerofoil to the trailing edge.
- The **angle of attack** which is generally known as α is the angle between the chord line and the earth's relative wind.
- The locus of points then become the mean **camber line** which joins the points between the upper surface and lower surface of Aerofoils.
- The resulting curvature of the mean camber line gives the aerofoil its profile. Normal aerofoils generally have a slightly cambered body.

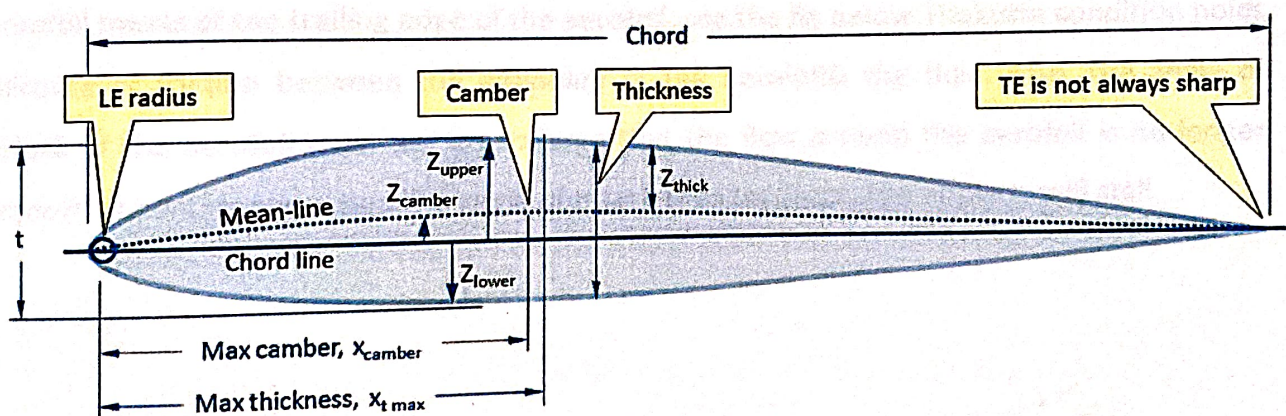


Figure 1: Aerofoil schematic

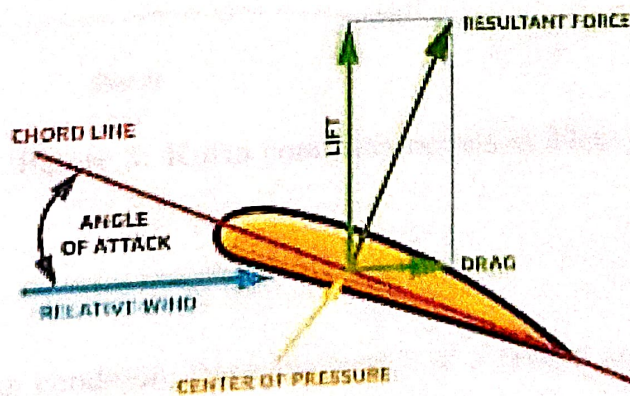


Figure 2: Force acting on aerofoil.

BERNOULLI'S PRINCIPLE-

We want to analyse the forces that act on an aerofoil when it is moving in the horizontal direction relative to the earth (ref – Figure 2), The aerofoil will experience a force R , which can be decomposed into two components namely the **drag D** in the direction of airflow and **L** which is normal to the flow. The lift is generally important for keeping the airplane flying, drag on other hand, is unwanted and has to be compensated by the propeller thrust.

To understand the reason behind the lift force L , we need to understand Bernoulli's principle. The most important part of the lift force experienced by the aerofoil is attributed mainly to the difference in atmospheric pressure at the upper and lower surfaces of the aerofoil. This difference in pressure at the upper and lower surfaces of the aerofoil results from the shape of the aerofoil & by assuming that the **kutta condition** holds.

KUTTA CONDITION:

The kutta condition states that the fluid flowing over the upper & lower surfaces of the aerofoil meets at the trailing edge of the aerofoil, see the fig below. The kutta condition holds because of friction between the boundary of the aerofoil & the fluid. Also, the angle of attack of the aerofoil must not be so large that the flow around the aerofoil is no longer smooth or continuous. When the angle of attack is so too large, the airplane will stall.

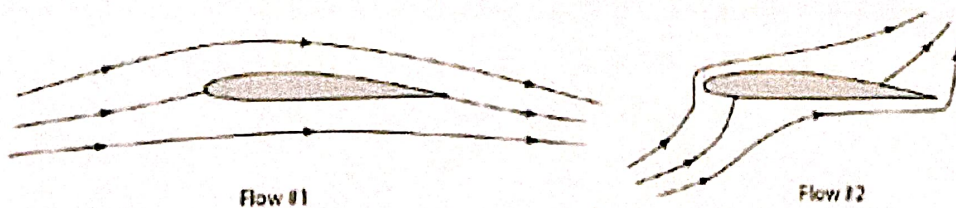


Figure 3: Kutta condition occurs in Flow # 1

Thus, because of kutta condition, since the length of a typical aerofoil's upper surface is greater, than the lower surface, the velocity of the fluid at the upper surface of the aerofoil is clearly higher than that at the lower surface. Bernoulli's principle states that an increase in

flow velocity for a fluid coincides with a decrease in the pressure, thus the fluid above the aerofoil will be less in fluid pressure as compared to surface below, which results in generating lift.

For steady motion of a fluid of density ρ along a body at rest, the pressure p at any point of the fluid of velocity U can be calculated from Bernoulli's equation which can be written as,

$$p + \frac{1}{2}\rho U^2 = p_0 + \frac{1}{2}\rho U_0^2$$

2- DIMENSIONAL FLUID DYNAMICS

There are two ways to proceed when studying the fluid interactions around an aerofoil. To make the distinction, we will need the following definitions.

Definition: Shear stress is a measure of the force of friction from a fluid acting on a body in the path of that fluid.

Definition: The viscosity of a fluid is the measure of its resistance to gradual deformation by shear stress. For example, honey is more viscous than water.

Here, we will assume:

- The fluid is **ideal** i.e. a fluid with zero viscosity.
- The fluid is **Incompressible** i.e. its density remains constant.
- The fluid must satisfy the **kutta** condition.
- The second is that we need the aerofoils velocity relative to surrounding fluid is to be **subsonic**, otherwise shock waves make the fluid flow discontinuous.

1. Potential Flow

Let us suppose an ideal fluid flowing over the ζ -plane with vector velocity U which, is horizontal and independent of depth. Let C be a curve in the ζ -plane passing through the points $\alpha = \alpha_1 + i\alpha_2$ and $\beta = \beta_1 + i\beta_2$, and let S be the surface through C with elements perpendicular to the ζ -plane. Let V_s denote the component of U along C at the point $\zeta = \xi + i\eta$, and let ξ and η components of U at ζ be u and v , respectively. Then, the flux across C through S is;

$$H = \int_{\alpha}^{\beta} -v d\xi + u d\eta.$$

Now H is independent of the path from α to β if and only if,

$$\frac{\partial(-v)}{\partial\eta} = \frac{\partial u}{\partial\xi}. \quad (1)$$

if (1) holds, the integral

$$\psi(\xi, \eta) := \int_{\alpha}^{\zeta} -v d\xi + u d\eta$$

defines a function ψ known as the stream function. If (1) holds, the fluid is incompressible. Again, equation (1) implies that the divergence of the fluid is zero. Thus, incompressible ideal fluid, the divergence of the fluid at every point is zero, which means that the fluid is not reduced in volume by an increase in pressure, and so the density of the fluid must remain constant. We define the circulation along C from α to β to be,

$$G = \int_{\alpha}^{\beta} V_s ds = \int_{\alpha}^{\beta} u d\xi + v d\eta.$$

Now, G is independent of the path from α to β if and only if

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \xi}. \quad (2)$$

If (2) holds, the integral

$$\phi(\xi, \eta) := \int_{\alpha}^{\zeta} u d\xi + v d\eta$$

defines a function ϕ known as the velocity potential. Notice here that equality of (2) implies that the curl of the fluid is zero. Thus, a fluid is said to be irrotational if (2) holds at every point. The complex function

$$F = \phi + i\psi = f(\zeta)$$

is called the generalized potential function. Since

$$\frac{\partial \phi}{\partial \xi} = \frac{\partial \psi}{\partial \eta} = u,$$

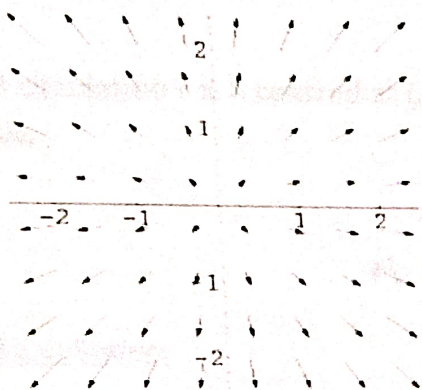
$$\frac{\partial \phi}{\partial \eta} = -\frac{\partial \psi}{\partial \xi} = v,$$

F is an analytic function of $\zeta = \xi + i\eta$. A flow that can be represented by such a function F is called a potential flow. Since F is analytic, the functions ϕ and ψ are harmonic functions. We may obtain the velocity components of F from $dF/d\zeta$. Thus,

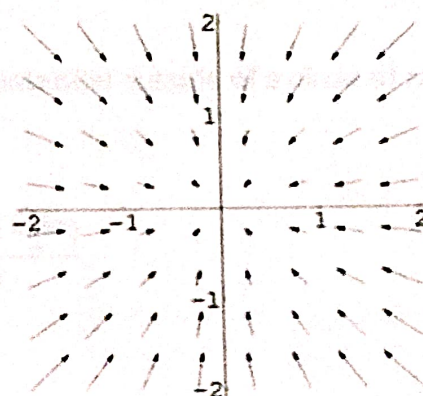
$$\frac{dF}{d\zeta} = \frac{\partial\phi}{\partial\xi} + i\frac{\partial\psi}{\partial\xi} = u - iv.$$

(3)

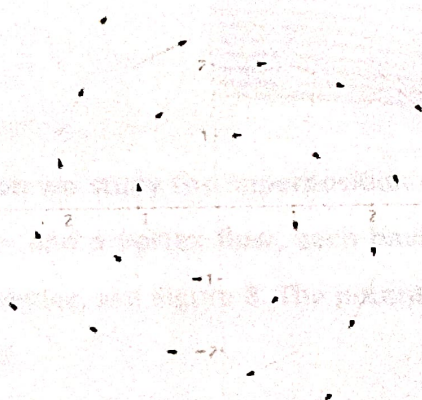
SOURCES, SINKS, VORTEX AND DOUBLET:



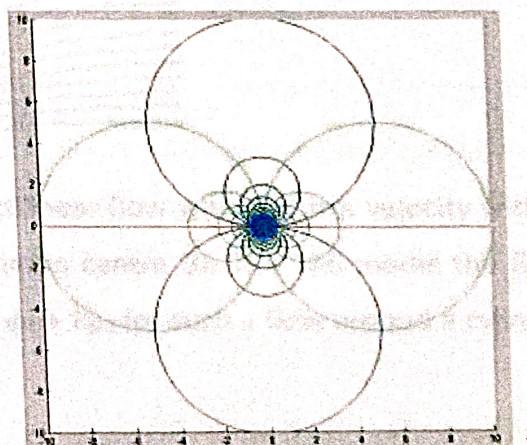
Vector field for a source flow.



Vector field for a sink flow.



Vector field for a vortex flow.



Vector field for a doublet.

Finally, for a *doublet* whose axis is the line of angle α with the ξ -axis and of moment (strength) M at ζ_0 , the potential is

$$F_D = -\frac{Me^{i\alpha}}{\zeta - \zeta_0}.$$

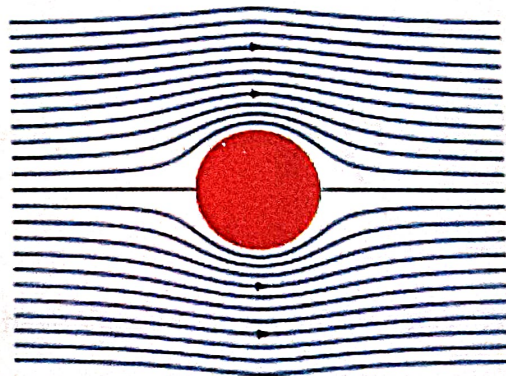
Consider a flow where all the streamlines are straight lines converging or diverging from a central point. If the flow is diverging from the central point, the flow is referred to as a source flow. and if the flow is converging to the central point, the flow is referred to as a sink flow. Here for fluid flowing with a uniform velocity $Ue^{i\alpha}$ at the point ζ_0 , where U and α are real, the potential function is,

$$F_R = \phi_R + i\psi_R = Ue^{-i\alpha}(\zeta - \zeta_0).$$

For a vortex of circulation $\Gamma \in \mathbb{R}$ centred at ζ_0 , see, the potential outside of a circle of radius a centered at ζ_0 is,

$$F_V = \frac{-i\Gamma \log\left(\frac{\zeta - \zeta_0}{a}\right)}{2\pi}.$$

Flow Around a Cylinder:



In this section we study the superposition of a rectilinear flow with uniform velocity with a doublet flow and a vortex flow, each having common centre ζ_0 . This will model the flow around a cylinder, see Figure 8. The potential $F_c = \phi_c + i\psi_c$ for such a flow around a cylinder of radius a is

$$F_c = F_R - F_D - F_V = Ue^{-i\alpha}(\zeta - \zeta_0) + \frac{Me^{i\alpha}}{\zeta - \zeta_0} + \frac{i\Gamma}{2\pi} \log\left(\frac{\zeta - \zeta_0}{a}\right). \quad (4)$$

Now, to properly model the flow around the cylinder we must require that the flow across the boundary of the circle $\zeta - \zeta_0 = ae^{i\theta}$ be zero. This means that for ζ on the circle, the imaginary part ψ_c of F_c must be zero, and from (4) it follows that $M = Ua^2$. The velocity function for this flow is given by,

$$\frac{dF}{d\zeta} = Ue^{-i\alpha} - \frac{Ua^2e^{i\alpha}}{(\zeta - \zeta_0)^2} + \frac{i\Gamma}{2\pi} \frac{1}{\zeta - \zeta_0} = U_\xi - iV_\eta. \quad (5)$$

By Bernoulli's equation, we have

$$\frac{\rho i}{2} \int_{|\zeta|=r} (U_\xi - iV_\eta)^2 d\zeta = p_\xi + ip_\eta = R,$$

where R is the total resultant force of the drag P_ξ and lift P_η , resulting from the pressure on the circular boundary $\zeta = re^{i\theta}$. If $\alpha = 0$, it follows that

$$p_\eta = 0, \quad p_\xi = \frac{\rho\Gamma U}{2} \left(1 + \frac{a^2}{r^2} \right).$$

Thus, if $a = r$, $P_\eta = \rho\Gamma U$. We have shown that if our circle of radius a represents the boundary of a cross-section of an infinity long, solid, cylindrical body placed in an in-viscous airstream, then the body will experience a per unit length lift force of measure $\rho\Gamma U$ at right angles to the stream. This result is known as the **Kutta-Joukowski** theorem.

METHODOLOGY

Suppose that, in the ζ -plane, there is a region G , which is the cross-section of a uniform, infinitely long body for which the potential function for a plane flow is known and is given by,

T ensures that the fluid velocity w_z at infinity in the z -plane is proportional to the fluid velocity w_ζ at infinity in the ζ -plane. We have $\mathcal{F}(z) = F(\zeta)$ when $\zeta = t(z)$, and thus it follows that for corresponding z and ζ , $\phi(\xi, \eta) = \Phi(x, y)$ and $\psi(\xi, \eta) = \Psi(x, y)$. Let $c_1, c_2 \in \mathbb{R}$. Then to each potential line $\phi = c_1$ in the ζ -plane there is a corresponding potential line $\Phi = c_1$ in the z -plane, and to each streamline $\psi = c_2$ in the ζ -plane there is a corresponding streamline $\Psi = c_2$ in the z -plane.

Now, condition (ii) on T that $\delta T / \delta z$ is finite and non-zero on the entire ζ -plane outside of G is made such that the family of curves $\phi = c_1$ and $\psi = c_2$ in the ζ -plane outside of G are mapped conformally, preserving angles, into the z -plane outside of P , forming the families $\Phi = c_1$ and $\Psi = c_2$. Since the curves $\phi = c_1$ and $\psi = c_2$ are orthogonal, the curves $\Phi = c_1$ and $\Psi = c_2$ must also be orthogonal. Thus, given a potential $F(\zeta)$ and a region G representing the cross-section of a uniform, infinitely long body, we have shown how to find the potential $F(z)$ for the flow of an ideal fluid around a body with any profile and cross-section given by P , which is formed from G by a conformal mapping.

APPLICATION

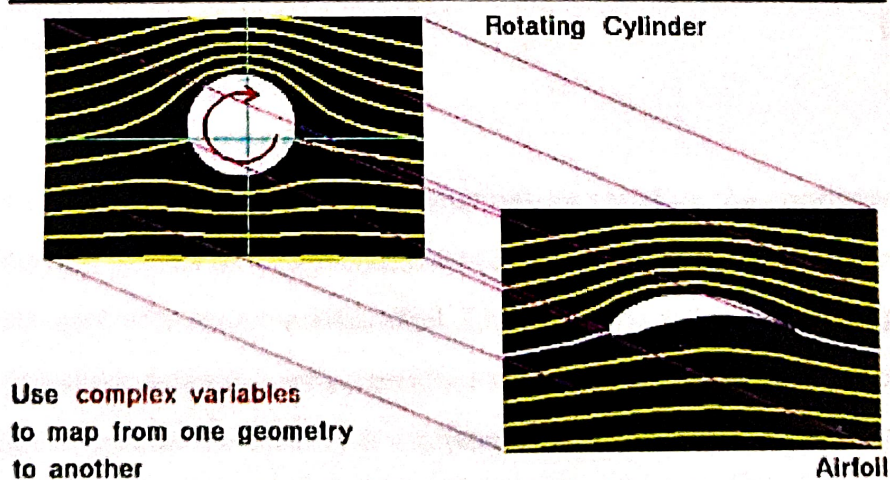
In this section we will think of G as being the cross-section of a cylinder of radius a and we will think of P as an aerofoil. We derived the potential function for flow around a cylinder. It is given by,

$$F(\zeta) = Ue^{-i\alpha}(\zeta - \zeta_0) + \frac{Ua^2e^{i\alpha}}{\zeta - \zeta_0} + \frac{i\Gamma}{2\pi} \log \left(\frac{\zeta - \zeta_0}{a} \right). \quad (9)$$



Conformal Mapping

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Now, we want our aerofoil P to satisfy the Kutta condition. Thus, we will assume that P has a "sharp trailing edge", and we will assume that the angle of attack α is small. In this case we can calculate the circulation Γ around the aerofoil and we can calculate $\frac{dF}{d\zeta}\bigg|_{\zeta_t} = 0$ to calculate the lift. Denote the trailing edge of the aerofoil P by $\bar{z}_t$ and the corresponding point on the cylinder G by ζ_t . We will orient ourselves such that the air foil P is facing left (like in the figures above). Then the trailing edge $\bar{z}_t$ of P occurs on the lower right-hand side of P . It will turn out that the corresponding point ζ_t of G will be on the lower right-hand side of the cylinder G and thus there is an angle $0 < \beta < \pi/2$ such that $\zeta_t = \zeta_0 + ae^{-i\beta}$

The Kutta condition is modelled mathematically by requiring from (5) and the Kutta condition, it follows that.

$$\frac{dF}{d\zeta}\bigg|_{\zeta_t} = Ue^{-i\alpha} - Ue^{i(\alpha+2\beta)} + \frac{i\Gamma}{2\pi a}e^{i\beta} = 0.$$

Thus,

$$\Gamma = \frac{2\pi a U}{i} \left(e^{i(\alpha+\beta)} - e^{-i(\alpha+\beta)} \right) = 4\pi a U \sin(\alpha + \beta). \quad (10)$$

By the

Kutta-Joukowski Theorem, the lift force acting on the aerofoil can be computed with the formula $\rho\Gamma U$. Thus, from (10) the lift force is given by $L = 4\pi\rho a U^2 \sin(\alpha + \beta)$. Now that we have developed all the theoretical background necessary to understand conformal transformations between cylinders and aerofoils, we present a couple of examples. We just need a transformation T , satisfying the conditions in section 4.1, mapping points from the z -plane to the ζ -plane. The first set of aerofoils are represented by the Joukowski transformation, which is given by,

$$z = \zeta + \frac{b^2}{\zeta}. \quad (11)$$

Given a properly defined inverse, this transformation satisfies the conditions in section 4.1. The non-conformal points of this transformation are the points $z = \pm 2b$ corresponding to the cusp at the end of a Joukowski aerofoil. Changing the value of b changes the shape of the resulting aerofoil. A small b value produces a thicker, cylindrical aerofoil. A larger value of b will create a thinner aerofoil. If $b = a$ (the radius of the cylinder in the ζ -plane), then the corresponding aerofoil will be a flat line, it is just the chord of the aerofoil. The

Joukowski family of aerofoils resulting from (11) have two important restrictions. One is that these aerofoils have zero tail angle at the trailing edge, and the other is that they are relatively thin.

The second transformation is a generalization of the Joukowski transformation. *The K'arm'an-Trefftz transformation* is given by,

$$z = \lambda \frac{\left(1 + \frac{b^2}{c}\right)^\lambda + \left(1 - \frac{b^2}{c}\right)^\lambda}{\left(1 + \frac{b^2}{c}\right)^\lambda - \left(1 - \frac{b^2}{c}\right)^\lambda},$$

where $\lambda = 2 - \theta / \pi$ and θ is the tail angle at the trailing edge of the aerofoil. Notice that if $\theta = 0$, a straight-forward, but messy calculation shows that the K'arm'an-Trefftz transformation reduces to the Joukowski transformation. The K'arm'an-Trefftz transformation yields aerofoils having improved fluid velocity and pressure distributions as compared to those of the Joukowski transformation.

Conclusion

Fluid flow around a wing is a very complex problem. But by reducing the problem to two dimensions and studying airfoils, it can be attacked using the complex variable techniques and in particular the geometric features of conformal mapping. There are plenty of Matlab scripts and Java applets on the net to help get intuition about the Joukowski and Kármán transformations.

Reference

- 1)Burington, R. S., On the Use of Conformal Mapping in Shaping Wing Profiles, The American Mathematical Monthly, Vol. 47, No. 6 (Jun. - Jul., 1940), pp. 362-373.
- 2)Conformal Mapping in Wing Aerodynamics Thomas Johnson June 4, 2013
- 3)Modeling the Fluid Flow around Airfoils Using Conformal Mapping Nitin R. Kapania, Katherine Terracciano, Shannon Taylor August 29, 2008
- 4)Foundation of Complex analysis, S. Ponnusamy